CITY SIZE AND DIVERSITY: OPTIMUM VERSUS LAISSEZ FAIRE

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City Size and Diversity: Optimum versus Laissez Faire

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Abstract

This paper, synthesizes a monopolistic competition setup portrayed in Dixit and Stiglitz (1977) with elements of urban economics, local public goods, and new economic geography (NEG) to explore the bias of laissez-fair city size and diversity vis-à-vis their optimum counterparts. Regarding both city size and diversity, the biase is ambiguous. However, in contrast to earlier studies, the paper finds that laissez-faire city size may be smaller than optimal city size and laissez-faire diversity may be larger than optimal diversity. Furthermore, only when laissez-faire city size is larger than optimal city size, laissez-faire diversity may be larger than optimal diversity. Using Cobb-Douglas utility function of leisure and differentiated goods, the paper shows that the relationships between laissez-faire city size and diversity and their optimal counterparts depends on aggregate population size. For sufficiently small aggregate population, optimal city size and diversity are larger than their laissez-faire counterparts; for intermmediate aggregate population, optimal city and diversity are smaller than their laissez-faire

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counterparts; and for sufficiently large aggregate population, optimal city size is smaller and optimal diversity is larger than their laissez-faire counterparts.

1 Introduction

This paper explores the bias of laissez-faire city size and diversity vis-à-vis their optimal counterparts.[1] There are different approaches to model diversity (see Eaton and Lipsey (1997)). Here, we restrict our discussion to the approach suggested in Section II of Dixit and Stiglitz (1977) where the utility increases with diversity, represented by a symmetrical CES function of brands' consumption. In addition, we assume that brands are produced in crowded cities, cities are established and run by profit-maximizing developers, and transporting brands between cities is costly, where transport costs reflect melting iceberg technology.

In this paper, we are mainly concerned with the interrelationship between city size and diversity and its implication on the bias of laissez-faire city size and diversity. With two exceptions that we are aware of, the issue of laissez–faire- versus optimal-city size, on the one hand, and laissez-faire versus optimal diversity, on the other, are discussed separately, where the city size issue is explored mainly in the urban economics literature.[2] The two exceptions are Henderson and Abdel-Rahman (1991) and Hochman (1997). However, in both studies, each city accommodates only one firm, such that diversity, measured by the aggregate number of firms, is proportional to the number of cities, which is proportional to the inverse of city size. This specification is not very appealing because our intuitive notion of diversity is associated with large cities rather than small ones. Furthermore, it cannot be justified unless the inter-city trade is costless. With costly inter-city trade, a centripetal force (saving resources used in inter-city trade) induces agglomeration of firms producing differentiated goods in a few large cities. In those circumstances, local diversity (number

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[1] We use the term "laissez faire" for economies in which the allocation and price system are determined by the market interplay of utility- and profit-maximizing agents. Thus, in our terminology, "laissez faire" need not exhibit price-taking or atomistic agents.

[2] See Tolley and Crihfield (1987) who claim that due to congestion and pollution, laisseez-faire large cities are too large, and Mills and Hamilton (1989), who challenge this hypothesis. See also Papageorgiou and Pines (2000), who discuss this debate and the confusionin involved in it.

of local firms) need no longer be constant, and, therefore, aggregate diversity (hereinafter diversity) is a weighted sum of local and non-local diversity.

We follow Henderson and Abdel-Rahman (1991) and Hochman (1997) by integrating the monopolistic competition setup of Dixit and Stiglitz (1977), on the one hand, and urban economics, including club and local public goods theories, on the other. In addition, we incorporate into the model the key element of the new economic geography (hereinafter NEG), first introduced by Krugman (1991): The costly intercity trade. Specifically, in building our framework, we borrow element from several sources: From Section II of Dixit and Stiglitz (1977), the utility function of the differentiated goods' quantity index and a numeraire, from Krugman (1991), the cost of inter-location trade, from urban economics literature (see, e.g., Mohring (1961)), the centrifugal force resulting from intra-city transportation cost, and from club and local public good theories (see, e.g., Tiebout (1956), Stiglitz (1977), and Henderson (1985)) the concepts of optimal city size and their laissez-faire realization through the activities of profit-maximizing developers.

It turns out that some of the result obtained by Henderson and Abdel-Rahman (1991) and Hochman (1997) are not robust enough to withstand the integration of a costly intercity trade into the analysis. Specifically, we show that when developers are minimally active (that is, they use only zoning and land revenue transfers), then

- *laissez-faire city size may be smaller* than optimal city size,
- if *laissez-faire city size is smaller* than optimal city size, then *laissez-faire diversity must be smaller* than optimal diversity,
- if *laissez-faire city size is larger* than optimal city size, then *laissez-faire diversity may be larger* than optimal diversity,

Using a Cobb-Douglas utility function of leisure and the quantity index of differentiated goods we illustrate the reversal of the Henderson and Abdel-Rahman (1991) and the Hochman's (1997) result. Indeed, when aggregate population size is either sufficiently large or sufficiently small, their result prevails; for an intermediate interval of city size, however, laissez-faire diversity may exceed that of optimal diversity.

The plan of the paper is as follows: Section 2 presents the setup of the paper that underlies both the optimum and the laissez faire regimes. Section 3 discusses and characterizes optimal allocations and the associated (implicit) price system. Section 4 elaborates the concept of laissez faire with profit-maximizing developers as well as the characteristics of laissez-faire allocations and the associated (explicit) price system. Section 5 compares the outcomes for local and global diversity under laissez faire and optimum. The results derived in section 5 are summarized in Section 6, where the robustness of the conclusions are discussed. The appendix, Section 7, provides explanations for some of the formulas reported in the text.

2 The Setup

$\mathcal{N}$ identical (in terms of both preferences and initial endowment) individuals are distributed among m cities (communities) with N_i in city i. The preferences of each individual can be represented by a well-behaved quasi-concave function of leisure (the numeraire), L, and a quantity index of differentiated goods, D:

$$u_i = u\left(L_i, D_i\right), \tag{1}$$

where the index of differentiated goods, D, is defined by

$$D_i \equiv \left[\int_0^B z^i\left(b\right)^{\frac{\sigma-1}{\sigma}} db \right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

B is the number (measure) of brands in the economy (degree of diversity), $z^i\left(b\right)$ is the consumption of brand b, and σ is the elasticity of substitution between brands. The output of each firm is viewed as a distinct brand. Each city i accommodates n_i firms such that $B = \sum_{i \in m} n_i$.

It takes $f + cx_i$ units of labor (the numeraire) to manufacture x_i units of a single brand in city i.

Due to identical technology across firms, identical preference across individuals, symmetry of brands in the utility function, and equal treatment of individuals in both the laissez-faire and normative analyses, individuals in any given city i consume an identical bundle of, leisure as well as locally produced and imported brands of differentiated goods. Accordingly, (2) reduces to

$$D_i = \left(n_i\left(z^{ii}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{j \in m, j \neq i} n_j\left(z^{ji}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where z^{ab} $(a, b \in m)$ is the consumption of a brand produced in city a by an individual who lives in city b.

Following the NEG tradition, an iceberg transportation technology is assumed. Accordingly, $\tau \geq 1$ units of a brand should be exported from its production location to allow consumption of one unit at the destination.[3,4]

With iceberg transportation technology, the material balance of a given brand produced in city i is given by

$$N_i z^{ii} + \tau \sum_{j \in m, j \neq i} N_j z^{ij} - x_i = 0. \tag{4}$$

Individuals are accommodated in linear monocentric cities. The width of each city is one unit of length. Each individual occupies one unit of land area. Accordingly, the city extends to a distance V_i from its center where

$$N_i - 2V_i = 0. \tag{5}$$

Each individual who lives v miles from the city center spends tv units of labor on commuting ($v \leq V_i$). Accordingly, the average distance commuted by each individual is $V_i/2$ and, therefore, the aggregate labor spent on commuting, ATC, is given by

$$ATC_i = N_i t \frac{V_i}{2} = t \frac{N_i^2}{4}. \tag{6}$$

Each individual living in city i supplies there I units of labor (the numeraire). Ac-

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[3] The iceberg "trick" proved to be very useful by considerably simplifying the analysis in its positive aspect. In particular, it implies uniform mill pricing, independently of the exported brand's destination, rather than price discrimination. This is so because, in this scenario, own-price elasticity and, consequently, marginal revenue are independent of transport cost. An exception to this approach is suggested in Ottaviano, Tabuchi, and Thisse (2000).

[4] In reality, the transport cost between pairs of cities may differ from one another, depending on their geographical distribution. In order to facilitate the analysis, we assume perfect symmetry.

cordingly, the labor constraint in each city is

$$n_i\,(f+cx_i)+N_iL_i+\frac{tN_i^2}{4}-N_iI=0. \tag{7}$$

3 Optimum

We consider a planner who chooses N_i, n_i, x_i, L_i, and z^{ij} to maximize the common utility, subject to $\sum N_i = \mathcal{N}$ and the resource constraints. Given the symmetry and concavity of the utility function as well as the identical technology available to existing and potential firms, maximization of uniform utility requires identical consumption bundles (L, D) for each individual. The optimum with such symmetry also requires that the cities be identical such that $n_i = n_j = n$, $x_i = x$, and $N_i = N$ for all $i,j \in m$. Thus, (3), (4), and (7) collapse to

$$D \;=\; n^{\frac{\sigma}{\sigma-1}}\left(\left(z^1\right)^{\frac{\sigma}{\sigma-1}}+\left(\frac{\mathcal{N}}{N}-1\right)\left(z^2\right)^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

$$N\left(z^1+\left(\frac{\mathcal{N}}{N}-1\right)\tau z^2\right)=x, \tag{9}$$

and

$$NL+n\,(cx+f)+\frac{tN^2}{4}-NI=0, \tag{10}$$

where z^1 and z^2 are the individual's consumption of locally produced and imported brands, respectively.

We portray determination of the optimal allocation as a two-step procedure. In the first step, the planner determines the optimal allocation for a given population size, N (number of cities, m); in the second step, he determines the optimal city size (number of cities).

3.1 Optimal Allocation for a Given City Size

Forming the Lagrangian corresponding to maximizing $u(L, D)$ subject to (8) $-$ (10) and deriving the first-order conditions with respect to z^1 and z^2,we obtain

$$z^{2\circ}(N) = z^{1\circ}(N)\,\tau^{-\sigma}, \tag{11}$$

where here and hereinafter, a variable designated by (N) denotes that it is conditional on N.

(11) allows us to further simplify (8) and (9) to

$$D^{\circ}(N) \;\;=\;\; z^{\circ}(N)\left(n^{\circ}(N)\,A(N)\right)^{\overline{\sigma-1}^{\,\sigma}} \tag{12}$$

and

$$NA(N)\,z^{\circ}(N) = x^{\circ}(N)\,, \tag{13}$$

where

$$z^{\circ}(N) = z^{1\circ}(N)\,.$$

and

$$A\left(N\right)\equiv 1+\left(\frac{\mathcal{N}}{N}-1\right)\tau^{1-\sigma}. \tag{14}$$

The other first-order conditions, together with (9), (10), (12), and (13), yield

$$x^{\circ}\left(N\right)=\frac{f\left(\sigma-1\right)}{c} \tag{15}$$

and

$$L^{\circ}\left(N\right)+D^{\circ}\left(N\right)P^{\circ}\left(N\right)=I-\frac{fn^{\circ}\left(N\right)}{N}-\frac{tN}{4}=E^{\circ}\left(N\right), \tag{16}$$

where

$$\begin{aligned} P^{\circ}\left(N\right) &\equiv \frac{u_D}{u_L}=\frac{cn^{\circ}\left(N\right)A\left(N\right)z^{\circ}\left(N\right)}{D^{\circ}\left(N\right)} \qquad (17) \\ &= c\cdot\left(n^{\circ}\left(N\right)A\left(N\right)\right)^{\frac{1}{1-\sigma}}=P\left(n^{\circ}\left(N\right)A\left(N\right)\right) \end{aligned}$$

and $E^{\circ}\left(N\right)$ is the minimum expenditure function conditional on N.

(13), (14), and (15) solve $z^{\circ}\left(N\right)$ in terms of the parameters

$$z^{\circ}\left(N\right)=\frac{1}{NA\left(N\right)}\frac{f\left(\sigma-1\right)}{c}. \tag{18}$$

Hence, using (12), we obtain

$$\begin{aligned} D^{\circ}(N) &= z^{\circ}(N)\left(n^{\circ}(N)\,A(N)\right)^{\frac{\sigma}{\sigma-1}} \\ &= \frac{1}{N}\frac{f(\sigma-1)}{c}\left(A(N)\right)^{\frac{1}{\sigma-1}}\left(n^{\circ}(N)\right)^{\frac{\sigma}{\sigma-1}} \qquad (19) \\ &= D^{\circ}\left(n^{\circ}(N), N\right). \end{aligned}$$

The remaining variables, $L^{\circ}(N)$ and $n^{\circ}(N)$, are to be solved by (16)-(19).

Three remarks are in order:

- (15) is a spatial version of the Dixit and Stiglitz (1977) result in which $x^{\circ}(N)$ is independent of N.
- (16) and (17) imply marginal cost pricing.
- $n^{\circ}(N)$, which is a measure of local diversity, resembles a public good.

In order to facilitate comparison between the optimal and the laissez-faire allocations in the sequel, we reformulate the planner's problem as using prices to manipulate consumer (compensated) demands. Accordingly, the planner chooses x, n, p^1, and p^2, to maximize u subject to (9) and (10) are adjusted by replacing z^1, z^2, and L by their compensated demands:

$$
\begin{aligned}
z^1: &\quad z^1\left(p^1, p^2, n, u\right), \\
z^2: &\quad z^2\left(p^1, p^2, n, u\right), \qquad (20) \\
L: &\quad L\left(p^1, p^2, n, u\right).
\end{aligned}
$$

Deriving the first-order conditions and substituting them into the adjusted resource constraints, we obtain $(11) - (19)$. Accordingly, (10) and (18) become:

$$NL\left(c, n^{\circ}(N) A(N), u^{\circ}(N)\right) + n^{\circ}(N) \sigma f = NI - \frac{tN^2}{4} \tag{21}$$

and

$$NA(N) z\left(c, n^{\circ}(N) A(N), u^{\circ}(N)\right) = \frac{f(\sigma - 1)}{c}, \tag{22}$$

respectively.
(22) and (21) can now be used to solve for $n^{\circ}(N)$ and $u^{\circ}(N)$. This completes the first step.

3.2 Optimal City Size (Optimal Number of Cities)

Turning to the second step, we explore the properties of $u^{\circ}(N)$,the characteristics of the optimal city size, and its determinants. To that end, we maximize $u(L, D)$ subject to $(16) - (19)$. Substituting (19) into $u(L, D)$ and $(17) - (19)$ into (16), the Lagrangian function can be reduced to

$$\begin{aligned}\mathcal{L} &= u\left(L^{\circ}(N), \frac{1}{N}\frac{f(\sigma-1)}{c}(A(N))^{\dfrac{1}{\sigma-1}}(n^{\circ}(N))^{\dfrac{\sigma}{\sigma-1}}\right) \\ + & \quad \lambda\left(L^{\circ}(N)+\frac{\sigma f n^{\circ}(N)}{N}+\frac{f n^{\circ}(N)}{N}+\frac{tN}{4}-I\right)\end{aligned} \tag{23}$$

Using the envelope theorem and substitute into the results the first-order condition with respect to $L^{\circ}(N)$, we obtain:

$$\begin{aligned}\frac{du^{\circ}(N)}{dN}=\frac{\partial\mathcal{L}}{\partial N} &= \frac{u_L}{N^2}\left(fn^{\circ}(N)-\frac{tN^2}{4}-fn^{\circ}(N)\frac{1}{A(N)}\frac{\mathcal{N}}{N}\tau^{1-\sigma}\right) \\ &= \frac{u_L}{N^2}\left(\frac{1-\tau^{1-\sigma}}{A(N)}fn^{\circ}(N)-\frac{tN^2}{4}\right).\end{aligned} \tag{24}$$

Substituting (10) and (15) into (24) yields

$$\text{Sign}\frac{du^{\circ}(N)}{dN}=\text{Sign}\left\{\left[\left(1-\tau^{1-\sigma}\right)\left(4(I-L)-t(1+\sigma)N\right)\right]-\sigma t\tau^{1-\sigma}\mathcal{N}\right\}. \tag{25}$$

Because the positive term inside the brackets of (25) is bounded from above by $4I$, it follows that:

Proposition 1 *(a) If $\mathcal{N}$ is sufficiently large, $u^{\circ}(N)$ is a decreasing function for all N and the optimal city size vanishes (formally becomes negative unless we impose a lower bound on city size. (b) If N is sufficiently large, $u^{\circ}(N)$ is a decreasing function of N.*

Proposition 1 corroborates the Anas (2004) result in a more general setup: The utility is a function not only of the differentiated goods index but also a function of leisure ; furthermore, the utility representation is not restricted to a specific functional form. The first extension is essential to distinguish between optimum and laissez faire that is the focus of our discussion.

One may wonder why the Anas (2004) result is robust to withstand inclusion of leisure in the utility function. The answer is provided by the Henry George theorem, which states that optimal economy size requires that the aggregate sum of implicit profits (the total implicit value of output minus the total value of inputs) disappears when both outputs and inputs are evaluated according to marginal-cost pricing (see Berglas and Pines (1981)). Because in our case the numeraire, that is, labor, is also directly consumed as leisure, the production function of leisure (using labor) exhibits constant returns to scale. Hence, the implicit profits associated with leisure vanishes. In consequence, leisure's inclusion or exclusion in the utility function does not affect the Henry George formula (24). The explanation for the vanishing cities in the restricted setup of Anas (2004) equally applies to our case.

Because (7) implies that n is bounded from above by $N^{\circ}I/f$, it, too, disappears when $\mathcal{N}$ increases sufficiently. Nevertheless, we are unable to make general statement regarding the evolution of global diversity, $n^{\circ}A^{\circ}$, as aggregate population increases. If, however, we require that each city accommodates at least one firm, that is, $n^{\circ} \geq 1$, global diversity should eventually increase as aggregate population size sufficiently increases.

In order to further characterize $u^{\circ}(N)$, differentiate (24) at any N, say $\overset{\smile}{\overset{\frown}{N}}$, where $u^{\circ}(N)$ assumes an extreme value, and substitute (24) into the result to obtain

$$\operatorname{sign} \frac{d^2u^{\circ}(N)}{dN^2}\bigg|_{N=\overset{\smile}{\overset{\frown}{N}}} = \operatorname{sign}\left(\eta_{n:N} + \frac{\frac{\mathcal{N}}{N}\tau^{1-\sigma}}{A(N)} - 2\right)\bigg|_{N=\overset{\smile}{\overset{\frown}{N}}} < \operatorname{sign}\ (\eta_{n:N} - 1)\Big|_{N=\overset{\smile}{\overset{\frown}{N}}}, \qquad (26)$$

where $\eta_{n:N}$ is the elasticity of $n^{\circ}(N)$. Hence, $\eta_{n:N} < 1$ is a sufficient condition for the concavity of $u^{\circ}(N)$ at $N = \overset{\smile}{\overset{\frown}{N}}$, that is for $u^{\circ}(N)$ to be single peaked. Let ϵ denote the marginal elasticity of substitution between L and D and

$$s^{\circ}\left(P^{\circ}\left(n^{\circ}\left(N\right),N\right)\right) \equiv D^{\circ}\left(n^{\circ}\left(N\right),N\right) \cdot P^{\circ}\left(n^{\circ}\left(N\right),N\right)/E^{\circ}\left(n^{\circ}\left(N\right),N\right), \tag{27}$$

that is, $s^{\circ}\left(\cdot\right)$ is the optimal share of D in the expenditure on L and D at $N = \overset{\smile}{\overset{\frown}{N}}$. It can then be shown (see Appendix 7.1) that:

Proposition 2 *If* $\epsilon \geq 1$ *or* $\epsilon < 1$ *and* $\sigma > 2 - fn^{\circ}\left(N\right)/\left(N\left(I - tN/4\right)\right)$, *then*

$$\left(\eta_{n:N} - 1\right)\Big|_{N=\overset{\smile}{\overset{\frown}{N}}} < 0$$

and, according to (26), $u^{\circ}\left(N\right)$ *is a single-peaked function.*

Finally, observe that (24) is reminiscent of the Henry George rule with unpriced congestion, where the aggregate land rent plus the warranted congestion toll revenue finance the cost of a congestion-prone collective good (see Arnott (1979) and Berglas and Pines (1981)). In the present case, n plays the role of a collective good, $tN^2/4$, the aggregate land rent (which, in a linear city, equals aggregate transportation costs), and $\mathcal{N} fn\tau^{1-\sigma}/\left(AN\right)$ can be viewed as the aggregate toll revenue that equals the net congestion effect of each individual multiplied by population size. The net congestion effect is the decline in utility (in terms of labor units) resulting from the loss of imported brands minus saving on spending, as the number of cities decreases. The decline in utility of each individual is

$$\frac{u_D}{u_L}\frac{\partial D}{\partial N} = -\frac{\sigma}{\sigma - 1}\frac{u_D}{u_L}\frac{D^{\circ}}{A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^2}\tau^{1-\sigma} \overset{(17),(15)}{=} -\sigma fn^{\circ}\frac{1}{N^{\circ}A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^2}\tau^{1-\sigma} \tag{28}$$

and the saving on spending on the differentiated goods is

$$-cn^{\circ}z^{\circ}\frac{\partial A}{\partial N}=cn^{\circ}z^{\circ}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma}\overset{(13)}{=}cn^{\circ}x^{\circ}\frac{1}{N^{\circ}A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma}\overset{(15)}{=}\left(\sigma-1\right)fn^{\circ}\frac{1}{N^{\circ}A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma}. \tag{29}$$

Hence, the net aggregate congestion effect of city size increase is

$$\begin{aligned} N^{\circ}\left(\frac{u_D}{u_L}\frac{\partial D}{\partial N}+cn^{\circ}z^{\circ}\frac{\partial A}{\partial N}\right) &= -\sigma fn^{\circ}\frac{1}{A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma}+\left(\sigma-1\right)fn^{\circ}\frac{1}{A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma} \qquad (30)\\ &= \frac{1}{A^{\circ}}\frac{\mathcal{N}}{\left(N^{\circ}\right)^{2}}\tau^{1-\sigma}. \end{aligned}$$

Finally, we have to multiply the expression on the RHS of (30) by N° to obtain implicit toll revenue in the second expression on the RHS of (24), that is, $fn^{\circ}\tau^{1-\sigma}\mathcal{N}/\left(A^{\circ}N^{\circ}\right)$.

4 Laissez faire

Unlike the previous section wherein we were concerned with the optimal allocation dictated by an omnipotent central planner, here we study a symmetric equilibria generated by utility-maximizing individuals, profit-maximizing manufacturers, and surplus-maximizing developers. Specifically, individuals, taking as given the prices of local and imported brands and the local and global diversity, choose their utility-maximizing consumption bundle and location, both across cities and location inside the chosen city. Monopolistically competitive manufacturers choose their production plan and prices which maximize their profits, taking as given demand for their brand. Developers determine city size through zoning, collect land rent, and redistribute part or all the rent revenue as head transfer to the local residents in order to maximize city surplus. The city surplus is the value of resources supplied in the city minus the value of resources used for leisure, production of manufactured goods, and commuting. In maximizing surplus, developers take the prevailing utility and prices, the number of cities, and the individuals' demands as given.

Migration is costless and so is the entry into and exit from the differentiated goods and city formation sectors where, in the latter case, there are sufficiently large potential sites to enable establishment of new cities. In consequence, equilibrium requires equal achievable utility, both across locations inside cities and across cities, zero profits in manufacturing a brand of differentiated goods, and zero maximized surplus in each city.[5]

Alternative laissez-faire regimes may be distinguished according to the menu of policy tools available to developers. Here, we discuss the basic regime where developers are relatively "passive", that is, they apply only a few policy tools.[6]

4.1 Characteristics of the Laissez-faire Regime for a Given City Size and Diversity

According to Section 2, the representative city is linear, extending to distance V on the two sides of a central point that is the commuter's origin or destination for all trips. A resident who lives v miles from the center spends tv on commuting and $R(v)$ on rent, such that total locational costs are $tv + R(v)$. Because there is no alternative use for land other than residents, $R(V) = 0$. Location equilibrium requires constant location costs, which implies

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[5] Among other things, this requires perfect replicability and divisibility of cities. By perfect replicability we mean a sufficiently large number of identical cities can be established; by perfect divisibility we mean that the set of cities is convex such that the number of cities can be any real number.

[6] This regime is referred to by Henderson and Abdel-Rahman (1991) as "unregulated competitive equilibrium".

(see Mohring (1961)):

$$R(v)+tv=tV=\frac{tN}{2}\Longrightarrow R(v)=t(V-v)=t\left(\frac{N}{2}-v\right). \tag{31}$$

It follows that aggregate land rent, ALR, is

$$ALR=2\int_{0}^{V}t(V-v)\,dv=tV^{2}=\frac{tN^{2}}{4}, \tag{32}$$

which equals aggregate transportation costs.

Each monopolistic competitive firm maximizes its profits by equating marginal revenue to marginal cost. It can be shown that, with a sufficiently large number of brands, the own-price demand elasticity of the ordinary demand for a representative brand is σ. Hence, the marginal revenue of a brand is $p(\sigma-1)/\sigma$ where p is the brand's mill producer's price. Since the marginal cost of a brand is c, profit maximizing requires

$$c=\frac{\sigma-1}{\sigma}p\Longrightarrow p^{\ell}=\frac{\sigma}{\sigma-1}c. \tag{33}$$

Here and in the following, a superscript ℓ denotes the laissez-faire value of the designated variable.

Free entry of profit-seeking firms eliminates the profit, implying

$$\begin{aligned} f+cx &= xp \overset{(33)}{=} x\frac{\sigma}{\sigma-1}c, \\ \Longrightarrow x^{\ell} &= \frac{f(\sigma-1)}{c}. \end{aligned} \tag{34}$$

Comparing (34) to (15), it follows that a brand's laissez-faire output is equal to its optimal level, as in Dixit and Stiglitz (1977).

There are three potential sources for an individual's income. First, individuals are endowed with I units of labor that can be consumed as leisure or spent on work and transportation. Second, there are profits from manufactured goods. However, under monopolistic competition, these profits vanish. The third source is land rent. It is assumed that the land originally belongs to the developer, who transfers a share of T to the local population, that is, each resident receives T times ALR/N.[7] Accordingly, the laissez-faire income, Y, of a representative individual is

$$Y(N) = I + T\frac{ALR}{N} = I + Tt\frac{N}{4} \tag{35}$$

(in terms of labor units) where use is made of (32).

Utility-maximizing individuals spend their income on leisure, locally produced and imported brands, housing, and transportation so that their minimum expenditure on leisure and differentiated goods is:

$$\begin{aligned} E\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) &\equiv L\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) + \frac{\sigma-1}{\sigma}c\mathcal{A}_i z\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) \\ &= Y - tv - R(v) \qquad (36) \\ &\overset{(31),(35)}{=} I + (T-2)\frac{tN}{4}, \end{aligned}$$

$L(\cdot)$ and $z(\cdot)$are the compensated demands for leisure and brands, respectively, $E(\cdot)$ is the

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[7] Alternatively, the land belongs to the local residents in equal shares and $1-T$ is the tax rate levied on land rent.

minimum expenditure function,

$$\mathcal{A}_i \equiv n_i + n_{-i}\left(m-1\right)\tau^{1-\sigma}, \tag{37}$$

and "$-i$" denotes "not i".[8]

In deriving (36), we take into account that the delivery price of imported brand is $\tau\sigma c/\left(\sigma-1\right)$, the minimum expenditure implies (11), and that clearing the markets for brands and trade balance require that the value of consumption equals the value of production, that is,

$$N_i \mathcal{A}_i z\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) = n_i x_i = n_i \frac{f\left(\sigma-1\right)}{c}. \tag{38}$$

4.2 Profit-Maximizing Developers and Laissez faire Equilibrium

Profit-maximizing developers establish cities by determining zoning, renting land parcels, and attracting residents by redistributing all or part of the rent revenue. The profits they earn is the of the city's surplus, that is, the value of resources supplied, NI, minus the value of resources used for leisure, production of manufactured goods, and commuting subject to resources constraints and balance of payment. Accordingly, the formal problem solved by

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[8] Without loss of generality, we assume that cities other than i are identical.

city i's developer is to choose n_i and N_i that maximize $\pi(n_i, N_i)$, given by

$$\begin{aligned}\pi(n_i, N_i) &\equiv N_i I - N_i L_i\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) - n_i\left(f + cx^{\ell}\right) - \frac{tN_i^2}{4} \\ &\overset{(38)}{=} N_i\left(I - L\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) - c\mathcal{A}_i z\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) - \frac{fn_i}{N_i} - \frac{tN_i}{4}\right).\end{aligned} \tag{39}$$

In maximizing N_i, developers take m and, therefore, $\mathcal{A}_i$ as given. Hence, a surplus-maximizing city size which is finite but smaller than $\mathcal{N}$ requires

$$\begin{aligned}\frac{\partial\pi(n_i, N_i)}{\partial N_i} &= I - L\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) - c\mathcal{A}_i z\left(\frac{\sigma-1}{\sigma}c, \mathcal{A}_i, u_i\right) - \frac{tN_i}{2} \\ &\overset{(39)}{=} \frac{1}{N_i}\left(\pi(n_i, N_i) + fn_i - \frac{tN_i^2}{4}\right) = 0.\end{aligned} \tag{40}$$

Free entry of developers and excess supply of potential sites for development eliminate profits in the development sector. More specifically, if the prevailing number of developed cities allows positive profits, new cities will be established when developers offer higher proportion of the rent revenue, T, as a head subsidy to their residents. This competition among developers continues until the utility increases to a level that precludes positive profits (the land rent revenue is fully distributed to local residents). Hence, laissez faire implies

$$\pi^{\ell}(\cdot) = 0 \Longrightarrow T^{\ell} = 1, \tag{41}$$

With (41), (40) reduces to

$$fn^{\ell} - \frac{t\left(N^{\ell}\right)^{2}}{4} = 0. \tag{42}$$

A laissez-faire symmetrical equilibrium with a non-trivial city size, that is, $\mathcal{N} > N^{\ell} > 0$, is a set $\left\{N^{\ell}, n^{\ell}, u^{\ell}, T^{\ell}, m^{\ell}\right\}$ that satisfies $N_i = N_{-i} = N^{\ell}, n_i = n_{-i} = n^{\ell}, u_i = u_{-i} = u^{\ell}$ and solves (36), (37), and (42).

Under laissez-faire allocation, $\mathcal{A}_i = n^{\ell}A^{\ell}$; therefore, (38) collapses to

$$N^{\ell}A^{\ell}z\left(\frac{\sigma-1}{\sigma}c, n^{\ell}A^{\ell}, u^{\ell}\right) = \frac{f(\sigma-1)}{c}. \tag{43}$$

In the following, we use (22) and (43) to compare the optimum with the laissez-faire regimes.

We have seen that optimal city size diminishes with aggregate population size $\mathcal{N}$. Does the laissez-faire city size too diminish with $\mathcal{N}$? The intuition suggests that this need not be the case because developers ignore the congestion that affects the optimum's calculation. This intuition is corroborated by analyzing the effect of aggregate population on laissez-faire city size in the case of homothetic utility function.

For laissez faire, (27), (17), and (19) become

$$s\left(P^{\ell}\right) = D^{\ell}P^{\ell}/E^{\ell}, \tag{44}$$

$$D^{\ell} = z^{\ell}\left(n^{\ell}A^{\ell}\right)^{\frac{\sigma}{\sigma-1}} = \frac{1}{N^{\ell}}\frac{f(\sigma-1)}{c}\left(A^{\ell}\right)^{\frac{1}{\sigma-1}}\left(n^{\ell}\right)^{\frac{\sigma}{\sigma-1}}, \tag{45}$$

and

$$P^{\ell}=\frac{\sigma}{\sigma-1}c\left(n^{\ell}A^{\ell}\right)^{\frac{1}{1-\sigma}}\overset{(42)}{=}K_1\left(\left(N^{\ell}\right)^2A^{\ell}\right)^{\frac{1}{1-\sigma}}, \tag{46}$$

respectively, where

$$K_1=\frac{\sigma}{\sigma-1}c\left(\frac{t}{4f}\right)^{\frac{1}{1-\sigma}}>0 \tag{47}$$

Using (36) and (41) ,we have

$$E^{\ell}=I-\frac{tN^{\ell}}{4}. \tag{48}$$

Substituting (42) , (45) , (46) , and (48) into (44) yields

$$s\left(P^{\ell}\right)=K_2\frac{4N^{\ell}}{4I-tN^{\ell}}, \tag{49}$$

where

$$K_2=\sigma f^{\frac{\sigma}{\sigma-1}}\left(\frac{t}{4}\right)^{\frac{1}{1-\sigma}}>0. \tag{50}$$

Differentiating (14) , (46) , and (49) with respect to $\mathcal{N}$ and solving for $dN^{\ell}/d\mathcal{N}$, we obtain

$$\frac{dN^{\ell}}{d\mathcal{N}}=-\frac{K_2P^{\ell}N^{\ell}\tau^{1-\sigma}s'}{(\sigma-1)\,A^{\ell}I^{\ell}s^2+K_2P^{\ell}N^{\ell}\left(A^{\ell}+1-\tau^{1-\sigma}\right)s'}. \tag{51}$$

It can be shown that *sign* $s'(P)$ =sign $(1-\epsilon)$ where ϵ is the elasticity of substitution of L for D in the utility (see Dixit and Stiglitz (1977)).[9] Hence, (51) implies:

Proposition 3 *Suppose that utility is homothetic. Then, (a) if the elasticity of substitution between L and D is smaller than one, implying that* $s'(P) > 0$, *then the laissez-faire city size,* N^{ℓ}, *diminishes with aggregate population size* $\mathcal{N}$; *(b) if the elasticity of substitution is unitary, that is,* $s'(P) = 0$ *(Cobb-Douglas utility function), the laissez-faire city size,* N^{ℓ}, *is independent of aggregate population size* $\mathcal{N}$.

5 City Size and Diversity

In this section, we assume that utility is homothetic. We also adopt the Dixit and Stiglitz (1977) condition which guarantees that the dd curve is more elastic than the DD curve, that is, $\eta_{s:P} + \sigma > 1$, where $\eta_{s:P} \equiv Ps'/s$ is the elasticity of s with respect to P (see equations (13) and (17) in Dixit and Stiglitz (1977)).[10] It turns out that this condition is more than sufficient to guarantee that nA and each brand are net substitutes, that is, $\partial z/\partial(nA) < 0$.[11] We further restrict the analysis to single-peaked $u^{\circ}(N)$ (see Proposition 2).

5.1 City Size

Consider the case where utility is represented by Cobb-Douglas function and σ and τ are are finite. Then, it follows from propositions 1 and 2 that, for sufficiently large $\mathcal{N}$, $N^{\circ} < N^{\ell}$, as in Henderson and Abdel-Rahman (1991) and Hochman (1997). This, however, need not always be true. Consider a case where $\mathcal{N}$ is finite and either σ or τ is sufficiently large, or

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[9] Be aware, however, of the different notation: $1/(1-\rho)$ and σ in Dixit and Stiglitz (1977) are denoted by σ and ϵ, respectively in our paper.

[10] $\theta(q)$ in Dixit and Stiglitz (1977) is denoted by $\eta_{s:P}$ in our paper.

[11] Net substitution between z and nA requires $\sigma + \eta_{s:P} > 1 - s$ (see Appendix 7.2), which is a weaker restriction than $\sigma + \eta_{s:P} > 1$.

σ or τ are finite and $\mathcal{N}$ is sufficiently small (only a single city can prevail under both laissez faire and optimum), such that the congestion term in (24) can be ignored. We will show that, in both cases, $N^\circ > N^\ell$.

First, we Prove that for any given N, $n^\circ(N) > n^\ell(N)$. To that end, we use (14), (22), and (43) to obtain

$$z\left(c, n^\circ(N) A(N), u^\circ(N)\right) = z\left(\frac{\sigma}{\sigma-1}c, n^\ell(N) A(N), u^\ell(N)\right). \tag{52}$$

Because z and nA are net substitutes, we can state:

Proposition 4 *Consider any symmetric population distribution with m cities, each of size N. Then, $n^\circ(N) > n^\ell(N)$.*

Proof. *Suppose not, that is, for some N, $n^\circ(N) \leq n^\ell(N)$ and (52) are satisfied. Then, because optimal price is lower, optimal utility is higher, and optimal diversity is lower than their laissez-faire counterparts, and because compensated demand decreases with price, increases with utility, and decreases with diversity, nA, it must be the case that $z\left(c, n^\circ\left(N^\ell\right) A\left(N^\ell\right), u^\circ\left(N^\ell\right)\right) > z\left(\sigma/(\sigma-1)c, n^\ell A\left(N^\ell\right), u^\ell\right)$. Contradiction.* ■

Second, when either σ or τ are sufficiently large and $\mathcal{N}$ is finite, or σ or τ are finite and $\mathcal{N}$ is sufficiently small, such that the congestion term in (24) can be ignored, we can write:

$$sign \lim_{\tau\to\infty} \frac{du^\circ(N)}{dN}\Big|_{N=N^\ell} = sign\left(fn^\circ\left(N^\ell\right) - \frac{t\left(N^\ell\right)^2}{4}\right) > sign\left(fn^\ell\left(N^\ell\right) - \frac{t\left(N^\ell\right)^2}{4}\right) = 0, \tag{53}$$

where the inequality follows from Proposition 4. (53) implies that $u^\circ(N)$ attains a local maximum at least, for a population size larger than N^ℓ. Because, by assumption, $u^\circ(N)$ is a single-peaked function of N, optimal city size exceeds that of laissez faire. This is illustrated in Figure 1.

Figure 1 here.

It follows, that in both cases, where the congestion term can be ignored, $N^{\circ} > N^{\ell}$. Hence, the discussion in this section demonstrates that, under our extended setup, optimal city size can be either larger or smaller (or even equal) to that of laissez faire, depending on the parameters.

5.2 City Size and Diversity

For the optimal and laissez-faire city sizes, (14), (22), and (43) yield

$$N^{\circ} A^{\circ} z\left(c, n^{\circ} A^{\circ}, u^{\circ}\right) = N^{\ell} A^{\ell} z\left(\frac{\sigma}{\sigma-1} c, n^{\ell} A^{\ell}, u^{\ell}\right) \tag{54}$$

(54) and the assumption that diversity and any single brand are net substitutes imply that

Proposition 5 *If $N^{\circ} \geq N^{\ell}$, then $n^{\circ} A^{\circ} > n^{\ell} A^{\ell}$.*

Proof. *Suppose not, that is, $N^{\circ} \geq N^{\ell}$ and $n^{\circ} A^{\circ} \leq n^{\ell} A^{\ell}$ are simultaneously valid. The premise, $N^{\circ} \geq N^{\ell}$, implies $N^{\circ} A^{\circ} \geq N^{\ell} A^{\ell}$ that together with (54), yields $z\left(c, n^{\circ} A^{\circ}, u^{\circ}\right) \leq z\left(\frac{\sigma}{\sigma-1} c, n^{\ell} A^{\ell}, u^{\ell}\right)$. However, the optimal price is lower, optimal utility is higher, and, by the premise, optimal diversity is not higher than their laissez-faire counterparts. Then, it must be the case that $z\left(c, n^{\circ} A^{\circ}, u^{\circ}\right) > z\left(\frac{\sigma}{\sigma-1} c, n^{\ell} A^{\ell}, u^{\ell}\right)$ because compensated demand decreases with price, increases with utility, and decreases with diversity, nA. Contradiction.*

■

It follows from propositions 4 and 5 that in the two cases discussed in the preceding section, those the congestion term in (24) can be ignored, that the optimal city is characterized by larger cities and more diversity than laissez-faire is. Thus, when transport costs are introduced, in contrast to Henderson and Abdel-Rahman (1991) and Hochman (1997), the larger optimal diversity than laissez-faire diversity is associated with larger optimal city size than laissez-faire city size.

Proposition 5 implies that laissez-faire diversity can exceed, if at all, that of the optimum, only if laissez-faire city size is larger than optimal city size. In the preceding section we have shown that this necessary condition can, indeed, be satisfied, particularly when τ and σ are finite and $\mathcal{N}$ is sufficiently large. The question is whether laissez faire can also generate more diversity than optimum in this case. The answer to this question is positive. We use a Cobb-Douglas utility function to prove it.

5.3 Optimal versus Laissez-faire City Size and Diversity: Cobb-Douglas Utility Function

When utility is represented by a Cobb-Douglas utility function, that is, $u(L, D) = L^{\alpha} D^{1-\alpha}$, the optimal and laissez-faire solutions for N and n are (details of their derivation are found in Appendix 7.3):

$$\begin{aligned}
N^{\circ} &= \frac{1}{1+\sigma-2\alpha}\left(\frac{4}{t}(1-\alpha)I\right)-(\sigma-\alpha)\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\mathcal{N}, \\
N^{\ell} &= \frac{4}{t}\frac{1-\alpha}{1+\sigma-\alpha}I, \\
n^{\circ} &= \frac{4}{tf}\left(\frac{1-\alpha}{1+\sigma-2\alpha}I\right)^{2} \\
&- \frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\left((\sigma-1)I\mathcal{N}+\frac{t}{4}(\sigma-\alpha)\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\mathcal{N}^{2}\right), \\
n^{\ell} &= \frac{4}{tf}\left(\frac{1-\alpha}{1+\sigma-\alpha}I\right)^{2}.
\end{aligned} \tag{55}$$

Thus, in accordance with propositions 1 and 3 optimal city size and local diversity decline with $\mathcal{N}$ and vanish altogether (formally, they become negative) as $\mathcal{N}$ exceeds a critical value, $\overline{\mathcal{N}}$, whereas laissez-faire city size and local diversity are positive and independent of $\mathcal{N}$.[12] Hence, there must exist another critical value of $\mathcal{N}$, say, $\underline{\mathcal{N}}$, such that for the interval $\left(\underline{\mathcal{N}},\overline{\mathcal{N}}\right)$, $N^{\ell} > N^{\circ}$.[13] However, when $\mathcal{N} > \overline{\mathcal{N}}$ the model collapses by generating negative values for N° and the comparison is meaningless. Anas (2004) solves the problem by requiring each city to accommodate at least one firm. We prefer to adjust the model by imposing a minimum city size, say δ, which prevails when $\mathcal{N}$ exceeds the level that generates δ, say $\widetilde{\mathcal{N}}$.

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[12] $\overline{\mathcal{N}} \equiv \dfrac{1-\tau^{1-\sigma}}{\tau^{1-\sigma}(\sigma-\alpha)}\dfrac{4(1-\alpha)}{t(1+\sigma-2\alpha)}I.$

[13] $\underline{\mathcal{N}} = \dfrac{1+\sigma-2\alpha}{1+\sigma-\alpha}\overline{\mathcal{N}}.$

Accordingly, the solutions for N° and n° in (55) are restricted to

$$\mathcal{N} < \widetilde{\mathcal{N}} \equiv \frac{1-\tau^{1-\sigma}}{\tau^{1-\sigma}(\sigma-\alpha)}\left(\frac{4(1-\alpha)}{t(1+\sigma-2\alpha)}I-\delta\right).$$

For $\mathcal{N} \geq \widetilde{\mathcal{N}}$, the corresponding values are

$$\begin{aligned} N^\circ &= \delta, \\ n^\circ &= \frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}\delta\left(I-\frac{t\delta}{4}\right). \end{aligned} \tag{56}$$

According to this modification, for $\mathcal{N} \geq \widetilde{\mathcal{N}}$, we have

$$n^\circ A^\circ = \frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}\left(I-\frac{t\delta}{4}\right)\left(1-\tau^{1-\sigma}\right)\delta+\frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}\left(I-\frac{t\delta}{4}\right)\tau^{1-\sigma}\mathcal{N} \tag{57}$$

Thus, we can fully compare optimal with that of the laissez-faire diversity. When δ tends to zero such that $\widetilde{\mathcal{N}}$ tends to $\overline{\mathcal{N}}$, we obtain:

$$\begin{aligned} n^\ell A^\ell &= \frac{4}{ft}\left(\frac{1-\alpha}{1+\sigma-\alpha}I\right)^2\left(1-\tau^{1-\sigma}\right)+\frac{1}{f}\frac{1-\alpha}{1+\sigma-\alpha}I\tau^{1-\sigma}\mathcal{N}, \\ n^\circ A^\circ\Big|_{\mathcal{N}<\overline{\mathcal{N}}} &= \frac{4}{ft}\left(\frac{1-\alpha}{1+\sigma-2\alpha}I\right)^2\left(1-\tau^{1-\sigma}\right) \\ &+ \frac{1}{f}\left(\frac{1-\alpha}{1+\sigma-2\alpha}\right)^2\left(2\tau^{1-\sigma}I\mathcal{N}+\frac{t}{4}\frac{\left(\tau^{1-\sigma}\right)^2}{1-\tau^{1-\sigma}}\mathcal{N}^2\right), \\ \lim_{\delta\longrightarrow 0} n^\circ A^\circ\Big|_{\mathcal{N}>\overline{\mathcal{N}}} &= \frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}I\tau^{1-\sigma}\mathcal{N}. \end{aligned} \tag{58}$$

Comparing the optimal values of N, n, and nA with their laissez-faire counterparts, we observe that:

1. For sufficiently small aggregate population size, $\mathcal{N}$, optimal city size is larger than laissez-faire city size (the intercept of $n^{\circ}A^{\circ}$ as a function of $\mathcal{N}$ is higher than that of $n^{\ell}A^{\ell}$). Consistent with Proposition 5, optimal diversity exceeds laissez faire diversity.
2. For sufficiently large population size, $\mathcal{N}$, say $\widehat{\mathcal{N}}$, optimal diversity exceeds laissez-faire diversity although optimal city size is smaller than laissez-faire city size (this follows from (58) , where $n^{\circ}A^{\circ}$ and $n^{\ell}A^{\ell}$ are linear in $\mathcal{N}$ but the coefficient of $n^{\circ}A^{\circ}$ as a function of $\mathcal{N}$ is larger than that of $n^{\ell}A^{\ell}$).
3. For sufficiently small aggregate population size, $\mathcal{N}$, the slope of $n^{\ell}A^{\ell}$ as a function of $\mathcal{N}$ is steeper than that of $n^{\circ}A^{\circ}$. Therefore, it is possible that the two functions intersect such that there exists an intermediate interval of $\mathcal{N}$ in which laissez-faire diversity exceeds optimum diversity.

To illustrate the possibility of case 3, we use an example based on the following set of parameters: $\alpha = .5$, $\sigma = 3$, $t = .2$, $\tau = 2$, $c = 1$, $f = 1$, $I = 10$. Figure 2 portrays the outcome of this specification. The solid curves refer to optimum and the dashed curves to laissez faire. In the upper panel, the bold solid curve represents optimal diversity for the interval $\mathcal{N} < \overline{\mathcal{N}}(= 120)$, whereas the solid thin curve represents optimal diversity for the interval $\mathcal{N} \geq \overline{\mathcal{N}}(= 120)$.

<u>Figure 2 here.</u>

According to Figure 2, one can discern five distinguishable intervals, the characteristics of which are summarized below:

$$\begin{array}{rccl}
\mathcal{N} & & & \\
[0, 17.1): & N^{\circ} > N^{\ell}, n^{\circ}A^{\circ} > n^{\ell}A^{\ell} & ; & z^{\circ} < z^{\ell}, \\
(17.1, 52): & N^{\circ} < N^{\ell}, n^{\circ}A^{\circ} > n^{\ell}A^{\ell} & ; & z^{\circ} > z^{\ell}, \\
(52, 120): & N^{\circ} < N^{\ell}, n^{\circ}A^{\circ} < n^{\ell}A^{\ell} & ; & z^{\circ} > z^{\ell}, \\
(120, \widehat{N}): & N^{\circ} = \delta < N^{\ell}, n^{\circ}A^{\circ} < n^{\ell}A^{\ell} & ; & z^{\circ} > z^{\ell}, \\
(\widehat{N}, \infty): & N^{\circ} = \delta < N^{\ell}, n^{\circ}A^{\circ} > n^{\ell}A^{\ell} & ; & z^{\circ} > z^{\ell}.
\end{array}$$

As reflected in Figure 2, optimal diversity exceeds that of laissez faire, that is, $n^{\circ}A^{\circ} > n^{\ell}A^{\ell}$, both in interval $\mathcal{N} \in [0, 17)$, where $N^{\circ} > N^{\ell}$, and in intervals $\mathcal{N} \in (17, 52)$, where $N^{\circ} < N^{\ell}$. However, laissez-faire diversity exceeds optimal diversity only in interval ($\mathcal{N} \in \left(52, \widehat{N}\right)$), where $N^{\ell} > N^{\circ}$. Note that the configuration portrayed in Figure 2 is not unique: With an appropriate set of parameters, $n^{\circ}A^{\circ}$ may be higher than $n^{\ell}A^{\ell}$ for all values of $\mathcal{N}$. Still, we can state:

Proposition 6 *There exists a non-trivial subspace of admissible utility functions and parameters for which laissez-faire city size and diversity exceed those of the optimum. With a Cobb-Douglas utility function, however, optimal diversity always exceeds laissez-faire diversity when aggregate population size is either sufficiently small or sufficiently large.*

6 Summary and Concluding Comments

In this paper, we synthesize the monopolistic competition setup portrayed in Section II of Dixit and Stiglitz (1977) with elements of urban economics, local public goods, and NEG to explore the bias of laissez-fair city size and diversity vis-à-vis their optimum counterparts. Earlier results derived by Henderson and Abdel-Rahman (1991) and Hochman (1997) showed that, at least for minimally active developers, optimum diversity is larger than laissez faire diversity, as in Section II of Dixit and Stiglitz (1977). Their observation, however, is restricted to a case where each city produces only one brand of differentiated good, thus equalizing diversity with the number of cities. This restriction can be justified only if inter-city trade is costless. We introduced inter-city trade cost into their model and reexamined the issue of laissez-faire versus optimal diversity and its relationship to the issue of laissez-faire versus optimal city size. Being a product of local diversity, n, and a function A, it turns out that more diversity may be associated with larger city size. In particular, when utility function is homothetic, larger optimal than laissez-faire city size is a sufficient condition for more optimal than laissez-faire diversity, while larger laissez-faire than optimal city size is a necessary but not sufficient condition for more laissez-faire than optimal diversity.

Using Cobb-Douglas utility function, we illustrated that the relationship between laissez-faire and optimal diversity crucially depends on aggregate population size. For both sufficiently small and sufficiently large aggregate population size, optimal diversity always exceeds laissez-faire diversity, as anticipated by Henderson and Abdel-Rahman (1991) and Hochman (1997). For some intermediate interval of aggregate population size, however, laissez-faire diversity may exceed that of the optimum.

The paper sheds some additional light on the debate regarding whether laissez-faire city size is too large. Most of the debate is concentrated on an urban setup with heterogeneous cities; the question is thus whether the market induces excessive inequality of city size distribution.[14] The present paper discusses a different setup, one where cities are identical. The question is whether laissez faire generates less than optimal number of cities (induces larger than optimal city size). There are two opposing effects that determine this relationship. On the one hand, the laissez-faire markup tends to reduce diversity that is a type of collective good. The smaller laissez-faire supply of this collective good reduces the advantage of cost saving associated with larger city size. On the other hand, the congestion effect of city size is ignored by developers who determine the laissez-faire city size; laissez-faire city size, therefore, tends to be larger than that of optimum. The relative importance of the second effect increases with aggregate population size and becomes dominant for sufficiently large aggregate population size. This is illustrated with the application of a Cobb-Douglas utility function. For small aggregate populations, the laissez-faire city size is smaller than that of the optimum, whereas for large aggregate populations, laissez-faire city size is larger than that of the optimum.

Our results were derived by using a very simple model that facilitates the analytical derivation of the main results in a quite general specification. First, like Henderson and Abdel-Rahman (1991) and Anas (2004) yet unlike Hochman (1997), we assume here that housing demand is perfectly inelastic with respect to both prices and income. Furthermore, in contrast to the two studies, we simplify the model by restricting the calculations to a linear rather than to a circular city. Second, our discussion is confined to the least active developers who do not interfere with the laissez-faire determination of local prices ("unregulated equilibrium", in Henderson and Abdel-Rahman's (1991) terms). We could have considered developers who control local mill prices (e.g., by subsidizing local production) or consumer prices of locally produced brands. We could also have considered alternative patterns of strategic behavior. We believe, however, that any extension along these lines

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[14] See Footnote 2.

cannot change the main result that, for an urban setting with costly inter-city trade, as in the NEG, laissez-faire may generate excessive rather than insufficient diversity. A more comprehensive characterization of the sufficient and necessary conditions for the outcomes of diversity in alternative allocation regimes is a challenge for future research.

Finally, in this paper we have ignored the issue of laissez-faire equilibrium stability. On first glance, it seems that the equilibrium portrayed in this paper is not stable because the surplus function has an inverted u shape. In this case, any accidental movement from city A to city B reduces the utility in both cities. Then, if individuals tend to move from a low to a high utility city, both cities begin losing population and, in a multi-city setup, the smaller city is eventually deserted. In order to solve this problem, we can modify the model by introducing some set up costs required to establish a new city. This modification implies that, at equilibrium, the developer's temporal profits are positive. In this case, a non-myopic developer whose city becomes smaller is willing to offer higher than average utility to his population in order to attract additional population. Indeed, in this case he suffers short run losses, yet he will do so whenever the discounted value of these losses is lower than that of his long run equilibrium profits afterwards. This modification may restore the original equilibrium. A full specification of such a model will be discussed in a separate paper.

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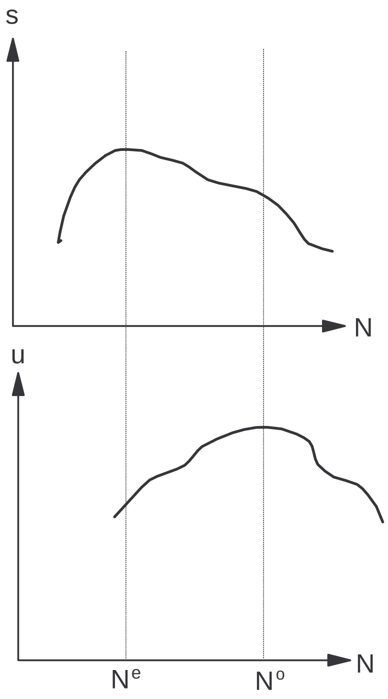

Figure 1

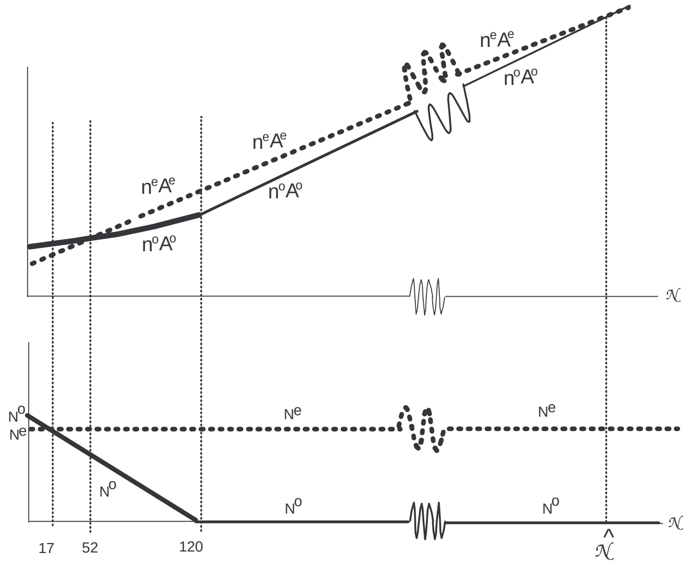
n^eA^e
n^oA^o
n^eA^e
n^oA^o
n^eA^e
n^oA^o
$\mathcal{N}$
N^o
N^e
N^e
N^e
N^o
N^o
N^o
$\mathcal{N}$
17
52
120
$\hat{\mathcal{N}}$

Figure 2

7 Appendix

7.1 Proof of Proposition 2:

Substituting (17) and (19) into (27) , one obtains

$$s\left(P\left(n^{\circ}\left(N\right),N\right)\right)E\left(n^{\circ}\left(N\right),N\right)=D^{\circ}\left(n^{\circ}\left(N\right),N\right)P\left(n^{\circ}\left(N\right),N\right)=f\left(\sigma-1\right)\frac{n^{\circ}\left(N\right)}{N}. \tag{59}$$

Differentiating (59) with respect to N yields

$$\eta_{n:N} = \frac{1+\eta_{s:P}\eta_{P:N}+\eta_{E:N}}{1-\eta_{s:P}\eta_{P:n}-\eta_{E:n}} \tag{60}$$

where $\eta_{a:b}$ is the elasticity of a with respect to b.

Evaluating the elasticities in (60) from (16) and (17) , we obtain

$$\begin{aligned}
\eta_{E^{\circ}:n} &= -\frac{fn^{\circ}\left(N\right)}{N}/E\left(n^{\circ}\left(N\right),N\right) &&= -\left(I-\frac{tN}{4}\right)/E\left(n^{\circ}\left(N\right),N\right),\\
\eta_{E^{\circ}:N} &= \left(fn^{\circ}\left(N\right)-\frac{tN^{2}}{4}\right)/E\left(n^{\circ}\left(N\right),N\right) &&= \left(I-\frac{tN}{2}\right)/E\left(n^{\circ}\left(N\right),N\right),\\
\eta_{P^{\circ}:n} &= \frac{1}{1-\sigma},\\
\eta_{P^{\circ}:N} &= -\frac{1}{1-\sigma}\frac{\mathcal{N}\tau^{1-\sigma}}{NA},
\end{aligned} \tag{61}$$

where $\eta_{a:b}$ is the elasticity of a with respect to b.

It can be shown that

$$\eta_{s:P} = (1-\epsilon)(1-s), \tag{62}$$

where $\eta_{s:P}$ is equivalent to θ in Dixit and Stiglitz (1977).

Substituting (61) and (62) into (60) yields

$$\eta_{n:N} = \frac{\left(I - \frac{tN}{2}\right)/E\left(n^{\circ}(N), N\right) + \frac{(\epsilon-1)(1-s)}{\sigma-1}\frac{\mathcal{N}}{NA}\tau^{1-\sigma}}{\left(I - \frac{tN}{4}\right)/E\left(n^{\circ}(N), N\right) + \frac{(\epsilon-1)(1-s)}{\sigma-1}}. \tag{63}$$

Inspecting (63), consider first the case where $\epsilon > 1$ globally. In this case, the second terms in both the numerator and the denominator are positive where the term in the denominator is larger than the term in the numerator. We also observe that the first expressions in both the numerator and the denominator are positive. Regarding the numerator, $I - tN/2 = I - tV$, is always non-negative because, otherwise individuals at the city boundaries use more resources than they contribute. The first term in the denominator is larger than the term in the numerator and, therefore, it is, a fortiori, positive. Hence, if $\epsilon > 1$, the numerator is positive and smaller than the denominator, implying $\eta_{n:N} < 1$.

Next, consider the case when ϵ is smaller than one and $\sigma > 2 - fn^{\circ}(N) / (N(I - tN/4))$. Since $(\epsilon - 1)(1-s) > -1$, the denominator of (63) is positive. Then, the numerator is either negative or positive. If it is negative, then $\eta_{n:N} < 0 < 1$. If it is positive, then the denominator exceeds the numerator and $0 < \eta_{n:N} < 1$. In both cases, however, $\sigma > 2 - fn^{\circ}(N) / (N(I - tN/4))$ implies $\eta_{n:N} < 1$. Q.E.D.

7.2 The effect of nA on $z(p, nA, u)$ for homothetic functions (Section 5)

We can rewrite (12) as

$$z(p, nA, u) = D(P, u)(nA)^{\frac{\sigma}{1-\sigma}}, \tag{64}$$

where $D(P, u)$ is the compensated demand for D and P is the price index of D, given by

$$P = p(nA)^{\frac{1}{1-\sigma}}, \tag{65}$$

where p is the price of z.[15]

Hence, when the utility is homothetic, (64) can be rewritten as

$$z(p, nA, u) = D(P, u)(nA)^{\frac{\sigma}{1-\sigma}} = \frac{s(P)}{P} E(P, u)(nA)^{\frac{\sigma}{1-\sigma}}, \tag{66}$$

Differentiating (65) and (66) with respect to nA and solving for $z_{nA}(p, nA, u)$, one obtains

$$z_{nA}(p, nA, u) = -\frac{1}{\sigma-1}\frac{z(\cdot)}{nA}(\eta_{s:P} + \sigma - 1 + s) \tag{67}$$

Hence, (66) implies

$$z_{nA}(\cdot) < 0 \Leftrightarrow \eta_{s:P} + \sigma > 1 - s, \tag{68}$$

which explains footnote 10.

---

[15] $p = c$ for optimum (see (17)) and $c\sigma/(\sigma-1)$ for laissez faire (see (46)).

7.3 Calculation of (55)

A Cobb-Douglas utility function, i.e., $u(L,D) = L^{\alpha} D^{1-\alpha}$, implies

$$L = \frac{\alpha}{1-\alpha} DP = \frac{\alpha}{1-\alpha} pnzA = \frac{\alpha}{1-\alpha}\frac{pnx}{N} = \frac{\alpha}{1-\alpha}\frac{pn}{N}\frac{f(\sigma-1)}{c}, \tag{69}$$

where use is made of (2), (13), (15), (17), (46), and (43).

Substituting (69) into (7) yields

$$n(N) = \frac{1-\alpha}{f}\frac{NI - \frac{tN^2}{4}}{(1-\alpha)\sigma + \alpha\frac{p}{c}(\sigma-1)}. \tag{70}$$

Because optimum implies $p/c = 1$ whereas laissez-faire implies $p/c = \sigma/(\sigma-1) > 1$, $n^{\circ}(N) > n^{\ell}(N)$ (an illustration of Proposition 3).

It follows from (14), (42), and (70), that

$$\begin{aligned}
N^{\ell} &= \frac{4}{t}\frac{1-\alpha}{1+\sigma-\alpha} I, \\
n^{\ell} &= \frac{4}{ft}\left(\frac{1-\alpha}{1+\sigma-\alpha} I\right)^2, \qquad (71) \\
n^{\ell}A^{\ell} &= \frac{4}{ft}\left(\frac{1-\alpha}{1+\sigma-\alpha} I\right)^2 \left(1-\tau^{1-\sigma}\right) + \frac{1}{f}\frac{1-\alpha}{1+\sigma-\alpha} I \tau^{1-\sigma}\mathcal{N}.
\end{aligned}$$

It follows from marginal cost pricing, (24), $du^{\circ}(N)/dN = 0$ (for $N = N^{\circ}$) and (70), that

$$N^{\circ} = \frac{1}{1+\sigma-2\alpha}\left(\frac{4}{t}(1-\alpha) I - (\sigma-\alpha)\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\mathcal{N}\right). \tag{72}$$

Hence, optimal city size vanishes as aggregate population size reaches $\overline{\mathcal{N}}$, which is given by

$$\overline{\mathcal{N}} \equiv \frac{4\left(1-\alpha\right)}{t\left(\sigma-\alpha\right)} I \frac{1-\tau^{1-\sigma}}{\tau^{1-\sigma}}. \tag{73}$$

Assuming that $N^{\circ}=\delta \Longrightarrow 0$ for $\mathcal{N} \geq \overline{\mathcal{N}}$, then

$$\begin{aligned} n^{\circ}\Big|_{\mathcal{N}<\overline{\mathcal{N}}} &= \frac{1-\alpha}{f\left(\sigma-\alpha\right)}\left(I-\frac{tN^{\circ}}{4}\right)N^{\circ} \\ &= \frac{4}{tf}\left(\frac{1-\alpha}{1+\sigma-2\alpha}I\right)^{2} \\ &- \frac{1}{f}\frac{1-\alpha}{1+\sigma-2\alpha}\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\left(\frac{t}{4}\frac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\left(\sigma-\alpha\right)\mathcal{N}^{2}+\left(\sigma-1\right)I\mathcal{N}\right). \\ n^{\circ}\Big|_{\mathcal{N}\geq\overline{\mathcal{N}}} &= \frac{1-\alpha}{f\left(\sigma-\alpha\right)}\left(I-\frac{t\delta}{4}\right)\delta. \end{aligned} \tag{74}$$

Using (74) and (14), it, then, follows that:

$$
\begin{array}{rcl}
n^\circ A^\circ\Big|_{\mathcal{N}<\overline{\mathcal{N}}} & & \dfrac{1-\alpha}{f\left(\sigma-\alpha\right)}\left(I-\dfrac{tN^\circ}{4}\right)\left(\left(1-\tau^{1-\sigma}\right)N^\circ+\mathcal{N}\tau^{1-\sigma}\right)\\[2ex]
 & = & \dfrac{1}{f}\left(\dfrac{1-\alpha}{1+\sigma-2\alpha}\right)^2\left(\dfrac{4}{t}\left(1-\tau^{1-\sigma}\right)I^2+2\tau^{1-\sigma}I\mathcal{N}+\dfrac{t}{4}\dfrac{\left(\tau^{1-\sigma}\right)^2}{1-\tau^{1-\sigma}}\right)\mathcal{N}^2\\[2ex]
n^\circ A^\circ\Big|_{\mathcal{N}\geq\overline{\mathcal{N}}} & = & \dfrac{1-\alpha}{f\left(\sigma-\alpha\right)}\left(I-\dfrac{t\delta}{4}\right)\left(\left(1-\tau^{1-\sigma}\right)\delta+\mathcal{N}\tau^{1-\sigma}\right)\\[2ex]
\lim\limits_{\delta\longrightarrow 0} n^\circ A^\circ\Big|_{\mathcal{N}\geq\overline{\mathcal{N}}} & = & \dfrac{1-\alpha}{f\left(\sigma-\alpha\right)}I\tau^{1-\sigma}\mathcal{N}.
\end{array}
\tag{75}
$$

When the parameters are $\mathcal{N}=100$, $I=10$, $f=1$, $c=1$, $t=.2$, $\alpha=.5$, and $\tau=2$, one obtains the optimum and laissez-faire allocations

$$
\begin{array}{rcccl}
N^\circ & = & \dfrac{1}{1+\sigma-2\alpha}\left(\dfrac{4\left(1-\alpha\right)}{t}I-\left(\sigma-\alpha\right)\mathcal{N}\dfrac{\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\right) & = & 5.556,\\
n^\circ & = & \dfrac{1}{f}\dfrac{1-\alpha}{\sigma-\alpha}N^\circ\left(I-\dfrac{tN^\circ}{4}\right) & = & 10.803,\\
A^\circ & = & 1-\tau^{1-\sigma}+\dfrac{\mathcal{N}}{N^\circ}\tau^{1-\sigma} & = & 5.250,\\
n^\circ A^\circ & & & = & 56.713,\\
N^\ell & = & \dfrac{4}{t}\dfrac{1-\alpha}{1+\sigma-\alpha}I & = & 28.571,\\
n^\ell & = & \dfrac{1-\alpha}{\sigma f}N^\circ\left(I-\dfrac{t}{4}\left(N^\circ\right)^2\right) & = & 40.816,\\
A^\ell & = & 1-\tau^{1-\sigma}+\dfrac{\mathcal{N}}{N^\ell}\tau^{1-\sigma} & = & 1.625,\\
n^\ell A^\ell & & & = & 66.326.
\end{array}
$$