MANIPULATIONS IN CONTESTS

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Abstract

We study the classical Tullock's model of one-stage contests where the probability of winning is a function of the efforts exerted by the contestants. We show that by a simple non-discriminating rule the contest designer is able to manipulate the outcome of the contest such that the probabilities to win are not ordered according to the contestants' abilities.

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JEL classifications: D72; L83

1 Introduction

In winner-take-all contests with a single prize, independently of success, all contestants bear the cost of their effort, but only one contestant wins the prize. In these contests, the designer can choose the contest architecture that will affect the outcome of the contest. For instance, she can determine whether the contest will be simultaneous or sequential (Gradstein and Konrad (1999)), the number of prizes (Moldovanu and Sela (2001)), and the number of contestants (Fullerton and McAfee (1999)) or the number of contestants at each stage of a multi-stage contest (Amegashie (1999)).

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In a one-stage contest where all the contestants compete against each other, if the contest rules are the same for all the contestants (non-discriminating rules), then, the designer is able to influence the absolute value of the winning probability of each contestant. However, usually she is not able to make a drastic manipulation such that the probabilities to win the contest are not ordered according to the contestants' abilities.[1]

In this paper we adapt Tullock's model (1980) to show that by a simple non-discriminating rule, the contest designer can determine the contestant with the highest probability. In particular, we show that in a two-player contest if the designer reimburses the winner's cost of effort, there is a unique internal equilibrium with undominated strategies where the weak contestant wins with higher probability than the stronger one. Moreover, in n player contests ($n > 2$), each one of the $n-1$ underdogs (all the players except the strongest one) may win with the highest probability, and the strongest contestant may choose to stay out of the contest when other contestants compete against each other.

The contest designers may have other goals in addition to determining the contestants' probabilities to win the contest. For example, in sporting contests (Szymanski (2003)) the designer may wish to maximize the total effort. On the other hand, in rent-seeking contests (Tullock (1980)), the designer may wish to minimize the total effort (total dissipation). If the contest designer wishes to maximize the total effort, it is well known that in the standard Tullock model the designer's expected payoff is smaller than the second highest value for winning the contest.[2] However, in our model, although the contest designer reimburses the cost of the winner's effort, her expected payoff, given that she maximizes the total effort, exceeds the second highest value. This paradox demonstrates that in some contests, depending on the designer's

[1] In elimination multi-stage contests, the contestants' probabilities of winning are not necessarily ordered according to the contestants' abilities, and these probabilities depend on the contest design (see, Groh et al. (2003)).

[2] See Baye et al. (1993, 1994,1996).

aim, it might be optimal to reimburse the effort cost of some contestants in order to increase their efforts and particularly to increase the contest designer's expected payoff in the contest.

2 Two player contests

Consider Tullock's model under complete information with two contestants. Each contestant's valuation for winning the contest is V_i $i = 1, 2$. Assume also that $V_1 > V_2$.[3] Every contestant exerts an effort x_i and the probability of winning for player i is

$$p_i(x_1, x_2) = \frac{x_i}{x_1 + x_2} \qquad i = 1, 2$$

The designer reimburses the winner's cost of effort such that the expected payoff of contestant i is

$$\pi_i = [V_i + x_i] \left[\frac{x_i}{x_1 + x_2} \right] - x_i \quad i = 1, 2 \tag{1}$$

Proposition 1 *There is a unique internal equilibrium in which the weak contestant (player* 2*) wins with higher probability than the strong contestant (player* 1*).*

Proof. Maximizing (1) with respect to the effort levels yields the following first order conditions:

$$\begin{aligned} \frac{\partial \pi_1}{\partial x_1} &= [V_1 + x_1] \left[\frac{x_2}{(x_1 + x_2)^2} \right] + \left[\frac{x_1}{x_1 + x_2} \right] - 1 = 0 \\ \frac{\partial \pi_2}{\partial x_2} &= [V_2 + x_2] \left[\frac{x_1}{(x_1 + x_2)^2} \right] + \left[\frac{x_2}{x_1 + x_2} \right] - 1 = 0 \end{aligned}$$

The unique internal solution of these equations is $x_1 = V_2$ and $x_2 = V_1$.[4]

Below we show that this equilibrium is unique. We split the strategy space to 4 sets and show that in each of these sets there are no other internal equilibrium points:

[3] A player with a higher valuation can be thought of as being more able.

[4] It can be verified that the equilibrium strategies $x_1 = V_2$, $x_2 = V_1$ are not dominated by other strategies.

1. $x_1 \leq V_2$, $x_2 \leq V_1$: At this set, contestant 2 has a dominant strategy of $x_2 = V_1$.

2. $x_1 > V_2$, $x_2 < V_1$: At this set every strategy of contestant 2, x_2, is dominated by the strategy $\widetilde{x}_2 < x_2$.

3. $x_1 \geq V_2$, $x_2 \geq V_1$: At this set contestant 1 has a dominant strategy of $x_1 = V_2$.

4. $x_1 < V_2$, $x_2 > V_1$: At this set every strategy of contestant 1, x_1, is dominated by the strategy $\widetilde{x}_1 < x_1$.

In addition to the unique internal solution in Proposition 1, there are some corner solutions in which one of the contestants chooses to stay out of the contest. For example if $x_1 > V_2$ then player 2 chooses to stay out of the contest and alternatively if $x_2 > V_1$ player 1 chooses to stay out of the contest. The contest designer can easily prevent these corner solutions by awarding the reimbursement to the winner only in the case of competition among the players, namely, if both players exert positive efforts.

We have shown that the reimbursement of the winner's cost of effort affects the contest outcome. Below we show that reimbursement of costs has another unexpected effect on the contest. Assume that the contest designer wishes to maximize the total effort. Then the revenue of the contest designer that reimburses the cost of the winner's effort is given by[5]

$$R = V_2 + V_1 - (V_2 \frac{V_2}{V_1 + V_2} + V_1 \frac{V_1}{V_1 + V_2}) = \frac{2V_2V_1}{(V_1 + V_2)}$$

It can be easily verified that the designer' payoff, independent of the contestants' valuations, is larger than V_2. It is important to note that although the designer reimburses the cost of the winner's effort, her payoff is larger or equal than the payoff in the imperfectly and perfectly discriminating cases of Tullock's model.

[5]It is assumed that the designer cannot derive any utility from the prize in the contest.

2.1 N player contests

Consider Tullock's model under complete information with $n > 2$ contestants. Each contestant's valuation for winning the contest is V_i, $i = 1, 2, ..., n$. Assume that $V_i > V_j$ for $i < j$. Similar to the case of two player contest, the winning probability and the payoff of player i are given by

$$p_i(x_1, x_2, ...x_n) = \frac{x_i}{x_1 + x_2 + ... + x_n} \qquad i = 1, 2, ...n \tag{2}$$

and

$$\pi_i = [V_i + x_i] \left[\frac{x_i}{x_1 + x_2 + ... + x_n} \right] - x_i \quad i = 1, 2, ...n \tag{3}$$

Proposition 2 *In the n player contest each one of the $n - 1$ underdogs (players $2, 3, ..., n$) may win with the highest probability.*

Proof. For simplicity we consider the case of three contestants. Maximizing (3) with respect to the effort levels yields the following first order conditions:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial x_1} &= [V_1 + x_1] \left[\frac{x_2+x_3}{(x_1+x_2+x_3)^2} \right] + \left[\frac{x_1}{(x_1+x_2+x_3)} \right] - 1 = 0 \\
\frac{\partial \pi_2}{\partial x_2} &= [V_2 + x_2] \left[\frac{x_1+x_3}{(x_1+x_2+x_3)^2} \right] + \left[\frac{x_2}{(x_1+x_2+x_3)} \right] - 1 = 0 \\
\frac{\partial \pi_3}{\partial x_3} &= [V_3 + x_3] \left[\frac{x_2+x_3}{(x_1+x_2+x_3)^2} \right] + \left[\frac{x_3}{(x_1+x_2+x_3)} \right] - 1 = 0
\end{aligned}$$

These equations have the following solutions:

1. $x_1 = V_2 \; x_2 = V_1, x_3 = 0$.

Given the analysis of a two player contest, it is sufficient to show that if player 1 exerts an effort of V_2 and player 2 exerts an effort of V_1, then player 3 has no incentive to enter the contest. Indeed, in this case $\pi_3 = \frac{x_3(V_3 - V_2 - V_1)}{x_1+x_2+x_3}$ is negative for all $x_3 > 0$. Note also that in this situation, player 2 wins with the highest probability.

2. $x_1 = V_3, x_2 = 0, \; x_3 = V_1$.

Again, it is sufficient to show that if player 1 exerts an effort of V_3 and player 3 exerts an effort of V_1, then player 2 has no incentive to enter the contest. Indeed, in this case $\pi_2 = \frac{x_2(V_2 - V_3 - V_1)}{x_1+x_2+x_3}$ is negative for all $x_2 > 0$. Note that in this situation player 3 wins with the highest probability. ⊡

Baye et al. (1993) looked at the optimal set of contestants in the perfectly discriminating case of Tullock's model (all-pay auctions), and found that it is sometimes advantageous to exclude the contestant with the highest valuation. In our model this may happen naturally as follows

Proposition 3 *In the n player contest the strongest contestant (player 1) may choose to stay out of the contest.*

Proof. Assume that player 2 exerts an effort of V_3 and player 3 exerts an effort of V_2. In this case the expected payoff of player 1 who exerts an effort of $x_1 > 0$ is given by $\pi_1 = \frac{x_1(V_1 - (V_2+V_3))}{x_1+x_2+x_3}$. If $V_1 < (V_2 + V_3)$ then player 1 will choose to stay out of the contest. ⊡

Propositions 2 and 3 show that if the strongest contestants (player 1) is much stronger than the other contestants ($V_1 > (V_2 + V_3)$) then there is no equilibrium where she is out of the competition. Otherwise, ($V_1 < (V_2 + V_3)$), the strongest contestant may not take part in the contest.

We have shown that, given the designer's goal, reimbursement of the winner's cost of effort may be profitable for the contest designer in the classical Tullock's model where the probability of winning is given by (2). It may be interesting to check the effect of reimbursement of the winner's cost of effort on other forms of Tullock's model, but these models as well as other models of contests are not necessarily solvable.

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