TRANSITIONAL DYNAMICS OF OUTPUT, WAGES AND PROFITS IN INNOVATION-LED GROWTH: A GENERAL EQUILIBRIUM ANALYSIS*

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ABSTRACT

Macroeconomic models that depict technological change as a progression of steady states are poorly suited to describing the singular diffusion of new technological paradigms such as the current revolution in information technologies. The present paper focuses instead on the *transition* between steady states of innovation-led growth, in terms of a model in which growth is triggered by the exogenous appearance of a new technological paradigm, and fueled by a wave of endogenous, stochastic, incremental innovations that implement the new paradigm. The analysis demonstrates existence of a unique Markov–perfect equilibrium in contingent consumption, production and development strategies, and shows that its transition dynamics conform in expected values to commonly observed empirical patterns. These include an initial productivity decline, followed by a greater increase in productivity; and "creative destruction" that reduces the market value of traditional, incumbent firms while creating new value in innovative entrants.

Keywords: growth, technology, innovation, transitional dynamics, general equilibrium
JEL categories: O41, O33

1 INTRODUCTION

Transitional dynamics play a key role in innovation–led growth. When technological change is ubiquitous, long–run steady states may be few and far between though short–run equilibria prevail in individual markets (Nelson and Winter, 1982). Detailed studies of major innovations describe the singular impact of specific "paradigm shifts" setting off waves of technological diffusion that last for decades before approaching a steady state (Freeman, Clark and Soete, 1982; Dosi, 1982).[1] Such waves have been associated with electric power (Freeman, 1982; David, 1991), crop hybridization (Griliches, 1957), synthetic fibers (Hollander, 1965) and semiconductors (Braun and MacDonald, 1982), each of which set off an extended diffusion process. More recently, Greenwood and Yorukoglu (1997) have argued from a macroeconomic perspective that the productivity slowdown in the 1970s was a transitional phenomenon that marked the beginning of a new industrial revolution based in Information Technology (IT).[2] Models of innovation-led growth that focus on steady–state outcomes are poorly suited to describing trajectories of technological progress such as these that are driven by the singular introduction of new technological paradigms.

The present paper directly addresses this issue by focusing on the transitional dynamics of innovation-led growth in the context of a general–equilibrium model. It models growth as triggered by the exogenous, random appearance of a radical innovation that challenges the ruling technological paradigm, and fueled by subsequent incremental innovations through which the economic impact of the new paradigm is diffused.[3] These incremental innovations are the stochastic result of endogenous investment in research and development by profit–maximizing firms.[4] The analysis demonstrates existence of a unique, Markov–perfect Nash equilibrium in contingent consumption, production and innovation strategies that describes a single trajectory of expected values, albeit with many possible realizations.

The shape of the wave typically follows a characteristic sigmoid pattern. Diffusion begins slowly because initially the new radical technology is not well–understood, and then gains momentum as "… the appearance of one or a few entrepreneurs facilitates the appearance of others, and these the appearance of more, in ever–increasing numbers" (Schumpeter, 1934, p. 228). Eventually, success breeds a surplus of economic capacity, which "… when in full swing … would eliminate entrepreneurial profit" (p. 235). This decline in incentives for innovative effort results in a fall in the rate of incremental innovation that persists until the next paradigm shift sets off a new wave of innovative activity.

Set in a general–equilibrium context that captures the macroeconomic effect of large–scale innovation, the transitional dynamics of the model are *analytically* shown to exhibit the following additional features:

- *A fall in measured productivity at the beginning of the wave*, mirroring empirically observed slowdowns in measured productivity at the beginning of the current IT revolution and, historically, in the first Industrial Revolution in Britain and in the early years of the American Industrial Revolution[5]. It occurs because resources are expended on research and development, to assimilate the new paradigm, before actual gains are realized; inasmuch as the assets created by this effort are intangible they do not directly contribute to conventionally measured output.[6] Subsequent productivity gains more than make up for this loss.
- *"Creative destruction" of established firms* that lack the agility to incorporate the new technological paradigm, and lose market value, while new value is formed in the start–ups that spearhead the technological revolution.[7] Recent examples of dramatic increases in the

market value of young IT companies overtaking the manufacturing giants of the past vividly illustrate this trend, which is also mirrored in the relative movement of the NYSE and NASDAQ indices at the beginning of the current IT revolution.[8]

- Two of Kaldor's (1966) stylized facts of growth: a secular increase in real wages, and trendless fluctuations in the distribution of income between wages and profits.

The model developed in this paper combines two important strands of the literature: stochastic partial–equilibrium models of strategic innovation,[9] and macroeconomic models of innovation–led growth.[10] By embedding a strategic model of innovation in a general equilibrium framework it adds structure and detail to the dynamics of innovation and diffusion, and introduces an aggregate stochastic dimension. Other recent efforts that similarly seek to model the specific dynamics of technological diffusion in a general–equilibrium setting include Andolfatto and MacDonald's (1998) extension of a real–business–cycle model in which technological shocks require costly effort before they are absorbed in the economy; Helpman and Trajtenberg's (1996) model of diffusion as the application of a "general purpose technology" (GPT) to the development of new product components; and Greenwood and Yorukoglu's (1997) analysis of the role of skills, capital vintages and learning–by–doing in the adoption of new technologies.

The present paper shares the general focus of these studies but goes beyond previous efforts in its explicit definition of a stochastic dimension that captures the essential uncertainty of innovative activity, and the indeterminate timing, duration and scope of the diffusion process at both the micro and macro levels. Furthermore, by demonstrating existence of a unique fully–specified stochastic equilibrium that describes a single trajectory of expected values, it provides a focal point for examining the dynamic macroeconomic properties of radical technological change. And it derives these properties analytically rather than by numerical simulation of selected trajectories, as previous efforts have largely done.

The structure of the paper is as follows: Section 2 defines the model. Section 3 demonstrates existence of a unique equilibrium. Section 4 analyzes its properties and illustrates them with a numerical example. Section 5 concludes with a brief summary.

2. DEFINITION OF THE MODEL

Assume a closed economy comprising a continuum of identical households of measure one deriving utility from consumption of a large number N of differentiated perishable goods. Each household inelastically supplies one unit of labor per unit of time and owns an equal share of each firm in the economy.

The list of goods is fixed, and each good is available at all times. Initially, these goods are produced through an existing technology by incumbent firms. When a new, radical innovation appears, it opens new opportunities for reducing the cost of supplying these goods through specific process improvements based on the new technology.[11] However, such improvements require further incremental innovation to assimilate the radical innovation in specific production processes, and new firms begin investing innovative effort to develop new production technologies for each of the N goods. These efforts succeed stochastically, each success allowing a reduction in the cost of an additional good, and continue until the scope for implementing the new technological paradigm is exhausted, or until a new paradigm appears.

We begin by describing the temporal equilibria of production and consumption at each stage of the diffusion process, before going on to characterize the dynamics of innovation.

2.1 Consumption and production

Let $x_i(t)$ denote the quantity of good i purchased and consumed by the representative household at time t, and let $p_i(t)$ denote its price. At any time t the representative household seeks to maximize the current expected value of its utility from future consumption

$$U(t) = \int_t^\infty [\sum_{i=1}^N x_i^\alpha(\tau)]^{1/\alpha} e^{-\beta(\tau - t)} d\tau \tag{1}$$

where $\beta > 0$ is a common rate of time–preference and $0 < \alpha < 1$ determines the elasticity of substitution between goods, $1/(1-\alpha)$. The intertemporal separability of household utility implies that the static allocation of consumption spending among the N goods at each point in time is independent of the stochastic–dynamic problem of allocating income between consumption and investment.[12] This allows us to begin by separately focusing on the temporal allocation of consumption spending among the N goods.

Production is achieved through the application of labor in fixed proportion to output (there is no physical capital in the model), where each producing firm's labor–to–output ratio is determined by its technical ability at the time of production. We assume that initially, at time $t = 0$, each good is produced by a single, incumbent, "old–technology" firm using a base technology that requires one unit of labor per unit of output; and we take the wage rate as our numeraire so that for all goods unit cost is equal to one.[13] At this time a new radical technology appears, and N innovating, "new–technology" firms, distinct from the incumbent firms, begin investing effort in adapting the new technology to the production of final consumption goods, each specializing in a different good. Firms that succeed in this effort—a discrete random event—achieve a process improvement that reduces the amount of labor required to produce the good. We assume for simplicity that for each good, within each paradigm shift, there is scope for only one incremental innovation resulting in the same proportional savings, $0 < \gamma < 1$; and that these savings are sufficiently large that the innovating firm entirely displaces the incumbent firm.[14] It then begins to recoup its investment in technology through its profit on sales in the product market. For notational convenience we assume without loss of generality that the goods are numbered in the order in which their cost of production is reduced.[15]

Let $n(t)$ denote the number of products produced at a unit cost of γ by new–technology firms at time t, while the remaining $N - n(t)$ goods are produced at a unit cost of 1 by old–technology firms. As production has no intertemporal dimension, each producing firm determines the price of its good so as to maximize current profits taking the price of other goods as given, so that the prices of the N goods are always in Nash–Bertrand equilibrium. For large N, the elasticity of demand for each good approximately equals $1/(1-\alpha)$, and so all producing firms set their markups equal to $1-\alpha$, to a close approximation. As the labor wage is the numeraire, the price P_0 of a good produced with the old technology and the price P_1 of a good produced with the new technology equal:

$$P_0 = 1/\alpha \tag{2a}$$

$$P_1 = \gamma/\alpha \tag{2b}$$

Straightforward derivation then shows that the utility–maximizing quantity of good i

purchased at time t by the representative household is fully determined by the number of goods produced using the new technology, $n(t)$, the level of current consumption spending, $c(t)$, and whether good i is being produced with the old or new technology. Denoting the utility–maximizing quantity of a good produced via the old technology by x_0, and of a good produced via the new technology by x_1, we have:[16]

$$x_0(t) = \delta_0(n(t))c(t) \tag{3a}$$

$$x_1(t) = \delta_1(n(t))c(t) \tag{3b}$$

where $\delta_0(n) = \alpha/(\gamma^{\alpha/(\alpha-1)}n + N - n)$ and $\delta_1(n) = \gamma^{1/(\alpha-1)}\alpha/(\gamma^{\alpha/(\alpha-1)}n + N - n)$. It follows that the temporal utility of the representative household also depends on t only through $n(t)$ and $c(t)$:

$$u(t) = [\sum_{i=1}^{N} x_i^{\alpha}(t)]^{1/\alpha} = [(N - n(t))\delta_0(n(t)) + n(t)\delta_1(n(t))]\ c(t) \tag{4}$$

Hence the marginal utility of spending on consumption is uniquely determined by n:

$$mu_n = \alpha\,(\gamma^{\alpha/(\alpha-1)}n + N - n)^{(1-\alpha)/\alpha} \tag{5}$$

The temporal profits of currently producing firms are similarly determined by $n(t)$ and $c(t)$. Denote the temporal profits of an old-technology firm by $\pi_0(t)$, and of a new-technology firm by $\pi_1(t)$. From equations (2) and (3), neither depends on the specific identity of the firm:

$$\pi_0(t) = (P_0 - 1)\, x_0(t) = (P_0 - 1)\ \delta_0(n(t))\ c(t) \tag{6a}$$

$$\pi_1(t) = (P_1 - \gamma)\, x_1(t) = (P_1 - \gamma)\ \delta_1(n(t))\ c(t) \tag{6b}$$

As all firms set the same markup, $1-\alpha$, this is also the share of profits in sales. And as each household supplies one unit of labor, and the labor wage is the numeraire, current household income equals $1 + (1 - \alpha)\, c(t)$. This must be allocated between consumption and investment in innovation.

2.2 Innovation and savings

Denote by $l_i(t)$ the rate at which firm i invests labor in its innovation program, which is also its rate of spending as the nominal wage identically equals one.[17] Following Schumpeter and many subsequent empirical studies we assume that there are positive spillovers from previous incremental innovations—through the movement of skilled workers between firms, demonstration effects, reverse engineering, etc.—which amplify firms' individual efforts. The probability of firm i's succeeding in its innovative efforts is therefore determined by the amount of resources it invests in innovation and by the previous success of other firms. To fix ideas, define for each innovating firm a cumulative variable, $z_i(t)$, that represents its stock of know-how at time t:

$$z_i(t) = \int_0^t b(n(\tau))m(l_i(\tau))d\tau \tag{7}$$

where b is an increasing function of n, and m is twice differentiable, strictly increasing and concave in the interval $[0,1/N]$ with $m(0) = 0$, $m`(0) = \infty$ and $m`(1/N) = 0$.[18] Success in this effort is a discrete random event, and we assume that the probability that firm i has successfully innovated by time t follows an exponential distribution with the cumulative distribution function $F(t) = 1 - \exp(-z_i(t))$. Hence the hazard rate of incremental innovation by firm i at time t (the statistical density of its successfully completing its innovation at that time conditioned on its not having done so earlier) equals the increment of its stock of know-how in that period:

$$(dF / dt) / [1 - F(t)] = dz_i(t) / dt = b(n(t))\, m(l_i(t)) \tag{8}$$

We assume that such knowledge does not carry over from one technological paradigm to the next. Therefore all innovating firms begin with a zero stock of know-how, and expect that if the next radical innovation appears before they have assimilated the current technological paradigm it will render obsolete all unembodied know-how.[19]

We further assume that management of each of the innovating firms is risk–neutral, and hence allocates the firm's resources so as to maximize its expected net present value discounted at a variable rate of return $r(t)$ determined by the market. Firms are not able to commit to future innovation levels; economic agents are assumed able to gauge theirs and others' probability of success from innovative effort, but are not expected to foresee actual future outcomes—who will succeed when; and all knowledge is common knowledge. Finally, we assume that at each stage of the innovation cycle all economic agents assign the same hazard rate $h(n)$ to the occurrence of a new paradigm shift, and share the same expectations regarding the current value of the income flow that firms which have successfully assimilated the current paradigm will earn in future cycles, which we denote $J(n)$.[20]

With regard to the allocation of income between consumption and savings, the additive separability of intertemporal utility implies that if $c(t)$ is an optimal consumption schedule then at any time t at which there is positive consumption, household savings are infinitely elastic at the interest rate that equates the current marginal utility of spending on consumption with the expected discounted marginal utility of consumer spending at any future time at which there is positive consumption.

3. EQUILIBRIUM

We now show that there exists a unique, symmetric, Markov–perfect Nash equilibrium in contingent innovation, production and consumption strategies that depend only on the stage of innovation n, and not explicitly on time.[21] In this equilibrium, households purchase consumption goods so as to maximize their expected intertemporal utility; producing firms set prices so as to maximize their profits taking the prices of other producing firms as given; innovating firms determine innovation spending so as to maximize the expected net present value of their cash flow taking the spending levels of other innovating firms as given; and product, labor and credit markets clear.

Specifically, we define a symmetric Markov–perfect Nash equilibrium to be a set of price levels and consumption quantities of old-technology and new-technology goods, respectively $P_0(n)$ and $P_1(n)$, and $X_0(n)$ and $X_1(n)$; consumption spending levels $C(n) = (N-n)P_0(n)X_0(n) + nP_1(n)X_1(n)$; interest rates $R(n)$; innovation investment levels $L(n)$; and value functions $V(n)$ and $W(n)$; all defined for $n = 0, \ldots, N$, such that at any time t when $n = n(t)$ firms have innovated:

1. Setting consumption quantities of old-technology and new-technology goods equal to $x_0(t) = X_0(n)$ and $x_1(t) = X_1(n)$ maximizes household temporal utility from consumer spending $c(t) = C(n)$, given prices $P_0(n)$ and $P_1(n)$.
2. Setting prices respectively equal to $P_0(n)$ and $P_1(n)$ maximizes the profits of old-technology and new-technology producers.
3. Setting innovation investment $l_i(t,n)$ equal to $L(n)$ maximizes the expected net present value at the interest rate $r(t) = R(n)$ of each of the $N-n$ currently innovating firms, given that each of the other $N-n-1$ innovating firms is also currently investing $L(n)$.
4. Setting consumption spending $c(t)$ equal to $C(n)$ maximizes households' expected intertemporal utility, given the interest rate $r(t) = R(n)$.
5. The expected current value of each innovating firm equals $V(n)$.
6. The expected current value of each currently producing new-technology firm equals $W(n)$.
7. The labor market is in full–employment equilibrium at each point in time.
8. Aggregate savings equal aggregate investment in innovation at each point in time.

Proof of existence and uniqueness is by recursive construction, proceeding by backward induction from the last stage of the diffusion process to the first. We first show that at stage N, when the new technological paradigm has been successfully assimilated in all sectors of the economy, there exists a unique set of stationary values that satisfy conditions (1)–(8) above. We then show—this is the induction hypothesis—that *if* at each stage $k > n$ there exists a unique set of stationary values that satisfy conditions (1)–(8) above *then* the same holds true for stage n . A heuristic proof is given below with further technical details provided in the Appendix.

Stage N. Consider a hypothetical time t before a new radical innovation has appeared, at which all N new firms have successfully assimilated the current radical innovation.[22] At this stage, all producing firms set the price of each good equal to $P_1 = \gamma/\alpha$. If consumption spending in equilibrium is $C(N)$ then total profits are $(1 - \alpha)\, C(N)$, and as the wage bill always equals 1, total income is $(1 - \alpha)\, C(N) + 1$. As the new paradigm has been fully assimilated, firms have no reason to invest in innovative effort and there is no demand for savings so total income must equal total consumption spending, $(1 - \alpha)\, C(N) + 1 = C(N)$, implying that $C(N) = 1/\alpha$ in equilibrium. If producing firms are maximizing profits, production of each good must equal $X_1(N) = (1/N)\, C(N)/P_1 = 1/(N\gamma)$. In the labor market, $L(N) = 0$ as there is no reason to invest effort in innovation, and demand for production labor is $\gamma N X_1(N) = 1$, which is labor supply, so that the labor market is in full employment equilibrium. As prices are stationary at this final stage of the cycle, and there is no investment demand, equilibrium in the credit market implies that $r(t) = \beta = R(N)$. It follows that the ENPV of future cash flows for each of the N firms, all of which are producing via the new technology, is $W(N) = [(P_1 - \gamma)\, \delta_1(N)\, c(N) + h(N)J(N)]/[h(N) + \beta]$, which does not depend on t.[23] There are no active innovating firms at this stage as there is no further scope for innovation, and for notational convenience we shall say that the ENPV of future cash flows for a notional innovating firm is $V(N) = 0$. Thus we have constructed a unique set of stationary values that satisfy conditions (1)–(8) above in the last stage of the innovation cycle.

Stage n < N. We proceed now by induction, to an earlier stage of the innovation cycle in which $n = n(t) < N$ firms have innovated successfully at time t, and the other $N - n$ firms are currently engaged in competitive investment in incremental innovation. We show that if the induction hypothesis holds for stages $k = n+1, \ldots, N$, then it holds also for stage n.

The $N - n$ innovating firms derive their profit maximizing levels of investment in innovation by simultaneously solving $N - n$ stochastic–dynamic programs. Assume that at time t, when n products are being produced with the new technology, firm i that has not yet successfully innovated, invests a constant stream of labor l_i for an infinitesimal time interval dt in research and development, conjecturing that the other $N - n - 1$ firms that have not yet innovated each invest l_k. Let $v_i(t,n)$ denote the expected current value of the cash flow of a firm currently engaged in developing new innovations, and let $w(t,n)$ denote the expected current value of the earnings of a currently producing new-technology firm.[24] Then we can say to a first-order approximation that the following holds:

- With certainty, it will pay out an immediate cash flow of $-l_i dt$.
- With probability $[1 - h(n)dt]\,[1 - \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\,]\cdot b(n)m(l_i)dt$ it will succeed in its efforts in this time interval while other firms do not, and radical innovation does not occur, so that the ENPV of its future cash flows will equal $w(t+dt,n+1)$ at time $t + dt$.
- With probability $[1 - h(n)dt]\,[1 - b(n)m(l)dt]\,\sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt$ radical innovation will not occur in this time interval, nor will firm i succeed in its efforts, but one of the other innovating firms will succeed, and so the ENPV of firm i's future cash flows will equal $v_i(t+dt,n+1)$ at time $t + dt$.
- With probability $[1 - h(n)dt]\,[1 - b(n)m(l)dt - \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\,]$, neither radical nor incremental innovation will occur in the interval, and the ENPV of firm i's future cash

flows will equal $v_i(t+dt,n)$ at time $t + dt$.

- With probability $h(n)dt$ radical innovation will occur in the interval $[t, t + dt]$, in which case we assume that a firm that has not yet innovated expects no future earnings.

Assume that firm i chooses its immediate level of investment so as to maximize the ENPV of its current and future cash flows, taking the conjectural rates of the other firms as given, and discounting end-of-period values at the current rate $r = r(t)$. Then, ignoring terms that are second order small (including the probability of simultaneous innovation) we obtain the following Bellman equation:[25]

$$v_i(t,n) = \lim_{dt\to 0}\max_{l\geq 0}\{ -ldt + b(n)m(l)dt\cdot w(t+dt,n+1) + \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\, v(t+dt,n+1) +$$

$$[1 - h(n)dt - r(t)dt - b(n)m(l)dt - \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\,]\, v_i(t+dt,n)\} \qquad (9)$$

for $i=n+1,\ldots,N$. Simultaneous solution of the $N-n$ dynamic programs defined by (9) determines the innovating firms' optimal investment levels in stage n as a function of the interest rate $r(t)$ and of the value functions $v_i(t,n+1)$ and $w(t,n+1)$ in the next stage. By the induction hypothesis we can rewrite (9) as:[26]

$$v_i(t,n) = \lim_{dt\to 0}\max_{l\geq 0}\{ - ldt + b(n)m(l)dt\, W(n+1) + \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\, V(n+1) +$$

$$[1 - h(n)dt - r(t)dt - b(n)m(l)dt - \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\,]\, v_i(t+dt,n)\} \qquad (10)$$

for $i=n+1,\ldots,N$. Letting $dt \to 0$, the profit maximizing investment strategy $l_i = l_i(t,n)$, and the value function $v_i(t,n)$ must then satisfy

$$- l_i(t,n) + b(n)m(l_i(t,n))W(n+1) + \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\, V(n+1) =$$

$$[h(n) + r(t) + b(n)m(l_i(t,n)) + \sum_{k=n+1,k\neq i}^{N} b(n)m(l_k)dt\,]\, v_i(t,n) \qquad (11)$$

The first–order condition that must obtain at an optimal solution of (11) provides a second equation for firm i, assuming it takes the conjectural rates of the other firms as given:

$$-1 + b(n)m`(l_i(t,n))[W(n+1)- v_i(t,n)] = 0 \qquad (12)$$

The strict concavity of m in the relevant range ensures that second–order conditions hold, and (12) uniquely determines $l_i(t,n)$ as a function of $v_i(t,n)$.

In a symmetric Nash equilibrium these equations must hold for each of the $N - n$ innovating firms. Dropping the subscripts from (11) and (12) we obtain the following two equations in the two unknowns $l(t,n)$ and $v(t,n)$, conditioned on the values of V and W in subsequent stages, and on the current interest rate $r(t)$:

$$- l(t,n) + b(n)m(l(t,n))W(n+1) + b(n)(N-n-1)m(l(t,n))V(n+1) =$$

$$[h(n) + r(t) + (N-n)b(n)m(l(t,n))]\, v(t,n) \qquad (13)$$

$$-1 + b(n)m'(l(t,n))[W(n+1)- v(t,n)] = 0 \qquad (14)$$

Equations (13) and (14) determine investment demand as a function of the interest rate. The additive separability of intertemporal utility implies that household savings are infinitely elastic at the interest rate that equates the marginal utility of spending on consumption at time t with its expected discounted marginal utility at time $t + dt$.[27] A necessary and sufficient condition for $r(t)$ to clear the credit market is therefore:

$$mu_n = [1 - r(t) - \beta] \{ [1 - (N-n)b(n)m(l(t,n))]\, mu_n + [(N-n)b(n)m(l(t,n))]\, mu_{n+1} \} \qquad (15)$$

where mu_n is the marginal utility of spending from (5), which depends only on n.

Equations (13), (14) and (15) are three equations in the three unknowns $r(t)$, $l(t,n)$ and $v(t,n)$. In the Appendix they are shown to have a unique solution under the conditions stipulated in Proposition 1 below, *viz.*, that N is sufficiently large and $m'/m'' \leq -m/m'$ for all $l \geq 0$.[28] As t appears explicitly in these equations only as an argument of the unknowns, this unique solution must be stationary within each stage, and we can denote its values $R(n)$, $L(n)$ and $V(n)$.

Having determined $L(n)$, the level of consumer spending in equilibrium must be such that the demand for production labor it generates results in full employment. As α is the share of labor in income from production, and the wage rate is the numeraire, the quantity of production labor must equal $\alpha c(t)$. Therefore the level of consumption spending that clears the labor market at stage n must satisfy

$$\alpha c(t) + (N-n)L(n) = 1 \qquad (16)$$

implying an equilibrium value of consumer spending, $C(n) = [1 - (N-n)L(n)]/\alpha$, that depends only on n.[29] Temporal utility is then maximized by consuming quantities of the different goods that are also stationary within each stage: $x_j(t) = \delta_j(n)C(n) = X_j(n)$ for $j = 0,1$.

The level of investment in innovation also determines $w(t,n)$, the ENPV of a firm currently producing via the new technology. Using (6b) to specify the profit stream π_1 of a new-technology firm:

$$w(t,n) = \lim_{dt\to 0}\{(P_1 - \gamma)\delta_1(n)c(t)dt + \sum_{k=n+1}^{N} b(n)m(l_k)dt\, w(t+dt,n+1) + J(n)\, h(n)dt + [1 - r(t)dt - h(n)dt - \sum_{k=n+1}^{N} b(n)m(l_k)dt\,]\, w(t+dt,n)\,\} \quad (17)$$

where, again, we omit from (17) terms that are second-order small. Setting $w(t+dt,n+1) = W(n+1)$ by the induction hypothesis, $l_k = L(n)$ for $k \geq n+1$, $c(t) = C(n)$ and $r(t) = R(n)$, we obtain also for $w(t,n)$ an expression that does not depend directly on t:

$$W(n) = [(P_1 - \gamma)\delta_1(n)C(n) + (N-n)b(n)m(L(n))W(n+1) + J(n)h(n)] \,/\, [R(n) + h(n) + (N-n)b(n)m(L(n)] \quad (18)$$

Thus we have constructed a unique set of stationary values that satisfy conditions (1)–(8) at stage n, which completes the proof. Summarizing[30]

Proposition 1*:* If N is sufficiently large, and $m'/m'' \leq -m/m'$ for all $0 \leq l \leq 1/N$, then there exists a unique symmetric Markov-perfect Nash equilibrium in contingent innovation, production and consumption strategies that depend only on the stage of the innovation cycle, that maximizes the expected discounted cash flow of firms and the expected discounted utility of households, and equilibrates supply and demand in goods, labor and credit markets. □

4. CHARACTERISTICS OF THE EQUILIBRIUM

The long run patterns generated by such a solution conform to several key empirical trends. This is shown analytically in Proposition 2, and then illustrated by a numerical example.

Property (*i*) in Proposition 2 describes an initial productivity slowdown and subsequent greater increase, which accords with the historical trends documented in Greenwood and Yorukoglu (1997) with regard to the Industrial Revolution in eighteenth century England and nineteenth century America, and the Information Revolution of the last twenty–five years.[31] The steady increase in wages (property *ii*) and the trendless variation of their share in national income (property *iii*) are facts 1 and 5 in Kaldor (1963); they imply an upward trend in profits that is not monotonic. The initial fall in market value of manufacturing firms and the concomitant creation of value in new innovating firms (property *iv*) mirrors similar trends in the NYSE and NASDAQ indices in 1975–79 discussed in note 7, above.

Proposition 2: If the current innovation cycle runs its course then:

(i) Measured productivity in the economy as a whole initially falls and then rises, eventually overtaking its level at the beginning of the current cycle, while the share of goods produced using the new technology rises monotonically.

(ii) The real wage rises monotonically over the course of the cycle; aggregate profits first fall and then rise more than they fell.

(iii) The share of wages in total output first rises and then falls, returning at the end of the

cycle to its starting level.

(iv) The market value of established manufacturing firms falls when a new radical innovation appears, while new innovating firms acquire value.

Proof: (*i*) If the preceding cycle runs its course before the current cycle begins then immediately before the beginning of the current cycle all labor is employed in production of consumption goods and using the previous technology, so total production is equal to one. Once the present cycle begins, some labor is employed in innovation, and so aggregate labor employed in production is less than one. Therefore, as no technological progress is immediately realized, aggregate production is also less than one, representing a decline in productivity because the total labor force is constant. If the current cycle runs its course, then when $n = N$ all labor will again be employed in producing final goods, but now using the new technology, and so total production will equal $1/\gamma$, which is greater than one. The monotonic diffusion of technology is inherent in the model.

(*ii*)–(*iii*) At each stage an additional sector switches to the new technology causing its output price to fall while other prices remain constant. As the nominal wage is constant the real wage continually rises. Total profits in the economy initially fall because prices are unchanged, output falls and the total wage bill—for production and innovation—remains constant (revenues fall while total costs are unchanged). Hence the share of wages in total product, equal to α just before the beginning of the present cycle, initially rises. If the cycle runs its full course the wage share again equals α in its last stage, implying that real profits must also eventually rise above their initial level.

(*iv*) As we have assumed that established firms cannot assimilate the new paradigm its appearance reduces the ENPV of their future cash flows, in anticipation of their gradual displacement by newly formed innovating firms. □

The following numerical example illustrates the initial drop in productivity, the movement of profits and wages, and variation in the value of firms described in Proposition 2, as well as a sigmoid pattern of diffusion of the new technological paradigm in the production sector. It is based on the following parameter values:

- The number of classes of goods is $N = 10$.
- The elasticity of substitution in consumption equals 2.5, i.e., $\alpha = 0.6$.
- Households' rate of time preference is $\beta = 0.1$
- The hazard rate of radical innovation is constant and equals $h = 0.0333$.
- Labor savings from the new technology are 25%, i.e., $\gamma = 0.75$.
- The knowledge-production function is $m(l) = l^{1/2}$
- The external benefits function is $b(n) = 1.4(1 + 1.3 \ln(n+1))$.
- New technology firms expect that the appearance of a new radical innovation will reduce the current expected value of their earnings by half, i.e., $J(n) = W(n)/2$ for $n = 1, 2, \ldots N$.

Based on these parameters a solution of the unique equilibrium values of $L(n)$ was computed recursively, allowing us to draw a diagram of expected levels of production at each stage of the long wave.[32] This is shown in Figure 1, which exhibits the initial productivity slowdown and subsequent greater rise implied by property *i* in Proposition 2.[33] (In this and the following figures the step function represents the original simulation data, and the continuous line connects the midpoints of adjoining steps.) Figure 2 shows the progression of wages and profits in real terms: wages exhibit a monotonic upward trend, and profits a cyclical upward trend (property *ii*); the share of wages in national income shows a trendless cyclical variation (property *iii*). Figure 3 shows the monotonic increase in the share of goods using the new technology in production (property *i*), which exhibits the characteristic sigmoid shape associated with technological diffusion. Figure 4 shows the initial fall in market value of manufacturing firms with an established history of sales and profits, and the concomitant creation of value in new innovating firms (property *iv*).

5. CONCLUDING REMARKS

The paradigm shifts and non-monotonicities that empirically characterize technological progress suggest that growth models that focus exclusively on steady–state outcomes cannot fully describe the macroeconomic dynamics of innovation–led growth. The present paper, by focusing on the economy's path in transition between steady states, within a general–equilibrium framework, provides an alternative perspective on technological progress. The model it puts forward describes a Schumpeterian cycle of growth that begins with the exogenous appearance of a new technological paradigm that is of no direct economic value itself, but which triggers a wave of incremental innovations through which the new paradigm is applied to the production of final goods. These incremental innovations are modeled as endogenous stochastic events that respond to the calculated efforts of rational entrepreneurs acting on profit opportunities.

The paper demonstrates existence of a unique Markov–perfect general equilibrium in contingent development, production and consumption strategies, which conforms in expected values to several observed empirical patterns. It implies an initial slowdown in measured productivity at the beginning of the technological revolution because of accelerated investment in intangible knowledge–based assets, mirroring similar productivity slowdowns observed historically and at the outset of the current revolution in information technology. It also indicates that established manufacturing firms—not as well–positioned as young innovative firms to assimilate the new technological paradigm—can be expected to lose value initially, while new value is created in successful startups. This is consistent with actual trends in the market value of firms. The model also replicates several of Kaldor's (1963) stylized facts of growth: a secular increase in productivity growth that more than compensates for the early deceleration, a steady rise in real wages, an upward trend in profits, and trendless cyclical movement in the functional distribution of national income. These dynamic patterns are analytically shown to hold in the model, and illustrated by a numerical example that also exhibits a monotonically increasing sigmoid pattern of diffusion in the application of the new technological paradigm to the production of final goods.

The present analysis suggests a feasible approach to integrating the transitional dynamics of technological revolutions within a general equilibrium framework that retains the explicit rationality and consistency in which neoclassical analysis is anchored. Furthermore, by demonstrating existence of a unique fully–specified stochastic equilibrium that describes a unique trajectory of expected values, it provides a focal point for analyzing technological change and technology policy in a macroeconomic context. Applications of this approach could be used to address the impact of technology policy on employment, investment and trade, and to analyze the implications of labor, credit and trade policies for technological progress.

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APPENDIX: PROOF OF PROPOSITION 1

Notation: Let $g(n; N) = [mu_{n+1}) - mu_n]/mu_n = [H(n+1;N)/H(n; N)]^{1/\alpha-1} - 1$, where $H(n; N) = \gamma^{\alpha/(\alpha-1)}n + (N-n)$.

Lemma: For given $0 < \alpha, \gamma < 1$, and fixed n, $g(n; N) \to 0$ as $N \to \infty$.

Proof: By substitution, $g(n; N) = \{1 + [\gamma^{\alpha/(\alpha-1)} - 1]/[(\gamma^{\alpha/(\alpha-1)} - 1)n + N]\}^{1/\alpha-1} - 1$ which is decreasing in n and therefore maximal at $n = 0$, and non-negative, so $0 \leq g(n; N) \leq g(0; N)$. As $N \to \infty$, $g(0; N) = [1 + (\gamma^{\alpha/(\alpha-1)} - 1)/N]^{(1-\alpha)/\alpha} - 1 \to 0$. □

Proposition 1: If N is sufficiently large, and $m'/m'' \leq -m/m'$ for all $0 \leq l \leq 1/N$, then there exists a unique symmetric Markov-perfect Nash equilibrium in contingent innovation, production and consumption strategies that maximizes the expected discounted cash flow of firms and the expected discounted utility of households, and equilibrates supply and demand in goods, labor and credit markets.

Proof: We prove existence by backward induction on n. The induction hypothesis is as stated in the text. Existence for $n = N$ is straightforward. For $n < N$, note first that for given $r(t)$, $\{l_k\}_{k \neq i}$, and $v_i(t+dt,n)$, there is a value of l that maximizes the RHS of equation (10), and because the maximand in (10) is continuous and $m'(l) = 0$ for $l \geq 1/N$, it satisfies $0 \leq l \leq 1/N$. Furthermore, as the maximand in (10) is non-negative for $l = 0$, at least for dt small enough, the maximal value of $v_i(t,n)$ must also be non-negative. Thus the correspondence that maps values of $v_i(t+dt.n)$ into maximal values of $v_i(t,n)$ must have a fixed point, and from the first order condition (12) it must be strictly smaller than $W(n+1)$. Furthermore, the envelope theorem implies that the slope of this correspondence is less than one so that this fixed point is unique. And as $W(n+1) > v_i(t+dt,n)$ for small dt the maximand is strictly concave in l, and therefore the maximizing choice of l must also be unique. Hence for any choice of $r(t)$ and

$\{l_k\}_{k\neq i}$ we have unique values of $v_i(t,n)$ and $l_i(t,n)$ that solve (10). In symmetric equilibrium this holds for all i, so equations (13) and (14) hold.

Next we show that there exists a unique choice of v, l and r that simultaneously satisfy equations (13), (14) and (15) and equilibrate demand for credit by innovating firms with its supply by households. Using (14) and (15) to substitute for $v(t,n)$ and $r(t)$ in (13) gives the following equation in l (dropping the time argument for convenience, and denoting as above $g(n) = [mu_{n+1}) - mu_n]/mu_n)$.

$$f(l) = -l + b(n)m(l)W(n+1) + b(n)(N-n-1)m(l)V(n+1) -$$
$$\{h(n) + \beta + (N-n)b(n)m(l)[1-g(n)]\}\{-1/[b(n)m'(l)] + W(n+1)\} = 0 \qquad \text{(A1)}$$

To see that (A1) has a solution in the interval $(0,1/N)$, note that when $l \to 0$ then $m(l) \to 0$ and $m' \to \infty$, and therefore $f(l)$ must be negative for small enough l; and when $l \to 1/(N-n) \geq 1/N$ then $m' \to 0$, so that the sign of $f(N-n)$ is determined by the sign of $h(n) + \beta - (N-n)b(n)m(1/N)[g(n) - 1]$, which must be positive for large enough N, from the Lemma. As f is continuous in l it must have a root in $(0,1/N)$ which induces values of $v \geq 0$ and r determined by (14) and (15).

To see that this solution is unique, take the derivative of f. Using abbreviated notation:

$$f'(l) = -1 - (N-n-1)(\mathrm{W(n+1)}-V(n+1))bm' + (N-n)g(n)W(n+1)bm'$$
$$-[(h+\beta)/b]m''/(m')^2 + (N-n)(1-g(n))[(m')^2 - mm'']/(m')^2] \qquad \text{(A2)}$$

If f has more than one root in the interval $[0,1/N]$ it must have a root where $f' < 0$. We show that if g is small enough this cannot hold. For $n = N-1$, (A2) reduces to:

$$f'(l) = -1 + g(n)W(n+1)bm' - [(h+\beta)/b]m''/(m')^2 + (1-g(n))[1 - mm''/(m')^2]$$
$$> -1 + g(n)W(n+1)bm' + 1 - g(n) + (1-g(n))mm''/(m')^2]$$
$$= g(n)bm'[W(n+1) - 1/(bm')] + (1-g(n))mm''/(m')^2 \qquad \text{(A3)}$$

which is positive if $g < 1$, as $W(n+1) - 1/(bm') = V(n) \geq 0$. For $n < N-1$, (A1) implies

$$[(N-n-1)(W(n+1)-V(n+1)) - (N-n)g(n)W(n+1)]b =$$
$$[-(h+\beta)W(n+1) + [(h+\beta)/b](1/m') + (N-n)(1-g)m/m' - l]/m \qquad \text{(A4)}$$

Substituting in (A2), we can write

$$f'(l) = -1 + (h+\beta)W(n+1)m'/m - (h+\beta)/(bm) - (N-n)(1-g(n)) + lm'/m$$
$$- [(h+\beta)/b]m''/(m')^2 + (N-n)(1-g(n))[1 - mm''/(m')^2]$$
$$= [(h+\beta)m'/m][W(n+1) - 1/(bm')] + lm'/m - [(h+\beta)/b]m''/(m')^2$$
$$+ (N-n)(1-g(n))[-mm''/(m')^2] - 1. \qquad \text{(A5)}$$

Noting again that $W(n+1) - 1/(bm') = V(n) \geq 0$, and $m'' \leq 0$, $f'(l)$ must be positive if

$(N-n)(1-g)[-mm''/(m')^2] > 1$, *i.e.*, if $1-g > 1/(N-n) \geq ½$, *i.e.*, if $g \leq ½$.

The proposition then follows from the Lemma, which shows that if N is large enough then g is arbitrarily small. □

The derivation in the Lemma implies that, for moderate values of α and γ, the requirement that "N is large enough" is not stringent. For example, if $\alpha, \gamma \geq ½$ then $N \geq 2$ is sufficient.

NOTES

[1] Various terms have been used to describe these radical innovations. "Paradigm shift" is an implicit reference to Kuhn's work on scientific revolutions. Bresnahan and Trajtenberg (1995) and Helpman and Trajtenberg (1996) refer more explicitly to "general purpose technologies". Other terms commonly used to describe new technological regimes are trajectories (Dosi, 1982) or filieres. Mokyr (1990) borrowed the term "punctuated equilibria" from evolutionary biology to describe the dynamics of technological development.

[2] In related work, Greenwood *et al.* (1997) point to a sharp increase in the relative importance of investment–specific technological change after the mid 1970s. Galor and Tsiddon (1997) similarly attribute the subsequent increase in the wage gap between skilled and unskilled workers to an acceleration of technological change.

[3] This distinction is grounded in Schumpeter's (1934) view of innovation and diffusion as fundamental phases of economic growth.

[4] Treating paradigm shifts as exogenous events is supported by Solomou's (1988) careful statistical investigation of long-run "phases of growth", which showed that historically such "episodic disturbances" have occurred randomly rather than at regular intervals.

[5] Greenwood and Yorukoglu cite Craft (1994) on an initial fall in productivity growth in the early stages of the Industrial Revolution in Britain (1760–1800), and Abramowitz and David (1973) on a slowdown in productivity growth in the early years of the American Industrial Revolution (1835–1855).

[6] We follow standard national accounting conventions in treating research and development (R&D) expenditures as an immediate expense rather than an investment in fixed assets. The 1999 decision of the U.S. Bureau of Economic Analysis to treat software development as fixed investment was an important step towards correcting this type of bias in national accounting (Moulton, 2000). A true measure of productivity that treated R&D spending as investment in a valuable though intangible asset would show a monotonic rise in productivity.

[7] Abernathy and Clark's (1985) notion of "transilience" speaks to this issue. Their analysis of the automobile industry distinguishes between "architectural" technological change, which undermines the position of the incumbent industry leader, and less radical change that can strengthen its position.

[8] Following Greenwood and Yorukoglu's (1997) timing of the current IT revolution as beginning in 1974, in 1975-79 the NYSE composite index of firms with an established history of revenues and profits fell by 3.8% in real terms while the NASDAQ index, with its more lenient listing requirements, registered a real increase of over 45% (U.S. Bureau of the Census, 1987, Tables 732, 807).

[9] Work in this vein includes Reinganum (1985), Jovanovic and Rob (1990), Jovanovic and Macdonald (1994), Justman (1996).

[10] Grossman and Helpman (1991) is a comprehensive early formulation of innovation-led growth. Subsequent work that modeled technological development as alternating phases of innovation and diffusion (or radical and incremental innovation) includes Aghion and Howitt (1992), Chou and Shy (1993), Cheng and Dinopoulos (1996) and Bental and Peled (1996). The present analysis extends their work by giving shape to the diffusion process.

[11] Although formally this is a model of process innovation, one can also think of the N goods as categories of needs—keeping time, relieving pain, preserving food—which are met with increasing efficiency by the introduction of better products. The model could also be formulated, with little change, as a model of explicit product innovation, with endogenous incremental innovation adding new products to the set of products that existed when the radical innovation appeared.

[12] Desired savings levels are determined by households equating the expected discounted marginal utility of consumption spending in all periods in which there is positive spending.

[13] The analysis could be predicated on asymmetric beginnings, with different firms initially using different production technologies. The symmetry of the model in this and other respects greatly simplifies the analysis, but is not generally essential for calculating a numerical solution.

[14] A sufficient condition for new firms to drive old firms out of the market is $\alpha > \gamma$, as will be made clear below. This is an extreme form of "creative destruction." Alternatively, we could assume—with little change in the formal analysis—that the new firm captures only some of the market, or that incumbent firms engage in assimilating the new paradigm.

[15] This order is not known *ex ante*, of course.

[16] Details of this and subsequent derivations are in the Appendix

[17] Innovation costs can be broadly construed here as including diverse categories of spending, e.g., laboratory research, testing, trial marketing, tooling, trial production, etc. An extension of the present model might have these stages appear in sequence.

[18] This description of innovation in the firm is based on the widely used partial–equilibrium models of competitive innovation developed by Kamien and Schwartz (1976), Reinganum (1982) and others. The model could equally accommodate negative external effects, e.g., if early innovators can delay innovation in other sectors by "defensive" patenting. In this case the function b in equation (7) would be decreasing in n. Concavity of m implies that accelerating the rate of innovation reduces its immediate efficiency. The last two assumptions on m\` ensure existence of an interior solution but are not essential (the second of these posits that the size of the entire workforce is larger than could be usefully employed in innovation.) The more realistic assumption that $m`(0) < \infty$ would allow for the possibility that some innovations are not worth pursuing at all. Alternatively, fixed costs could be introduced in the model as a precondition for incremental innovation that might rule out some innovations. Such fixed costs could also provide a basis for endogenizing the number of innovating firms.

[19] In other words, firms anticipate that current-cycle know-how not embodied in a specific product will be of no benefit after the next paradigm shift.

[20] Whether or not these commonly held beliefs actually materialize is not essential to the model; firms and households need not be able to accurately predict when the next paradigm shift will occur or what its implications will be. However, correct or not, their subjective beliefs about the next paradigm shift have a bearing on the shape of the current wave. A possible extension might have the subjective hazard rate of radical innovation change over time, e.g., with firms attaching higher values to p at early stages of the cycle, before the new paradigm is fully established; and at later stages, when it appears to have exhausted its usefulness.

[21] Competitive regimes in which firms are able to condition their strategies on the full history of past events would presumably allow a wider range of outcomes to be supported.

[22] The final stage may never be reached if a radical innovation cuts short the current cycle before it runs its full course; all the calculations are contingent calculations.

[23] $W(N) = \lim_{dt\to 0}\{(P_1 - \gamma)\, \delta_1(N)\, c(N)\, dt + J(N)\, h(N)\, dt + [1 - r(N)dt - h(N)dt]\, W(N)\,\}$

[24] As the temporal profits of a producing firm do not depend on the identity of the firm neither does their expected present value.

[25] This is an extension of Aldrich and Morton's (1975) use of Bellman equations in the single–firm case, and their application to the multi–firm case in Justman and Mehrez (1984).

[26] Note that when $n = N - 1$ the third term in the RHS of (10) is empty (summation is over other firms which have not yet innovated, of which there are none). Hence the value of $V(N)$ is immaterial, confirming that the notational convention $V(N) = 0$ is innocuous.

[27] Else there would be no consumption spending either at time t or at time $t + dt$, which cannot be an equilibrium (technically, because the innovating sector cannot profitably absorb all labor.)

[28] The condition $m'/m'' \leq -m/m'$ holds, *e.g.*, for functions of the type $m(l) = l^a$ if $a \leq ½$.

[29] To see that this implies that savings equal investment (an implication of Walras' Law) note that household income equals $1 + (1-\alpha)C(n)$. Savings are then obtained by subtracting consumption from income, giving $1 + (1-\alpha)C(n) - C(n) = 1 - \alpha C(n)$. After substitution for $C(n)$ this equals $(N - n)L(n)$, which is the level of aggregate investment in innovation at stage n.

[30] See the Appendix for further details of the proof. The condition on N depends on the values of α and γ, but is generally not stringent. For example, if $\alpha, \gamma \geq ½$ then $N \geq 2$ is sufficient, as shown in the Appendix.

[31] See notes 2 above. As the supply of labor is constant and capital does not appear explicitly in the model, the growth rate of real output to which Proposition 2 refers is also the growth rate of measured total factor productivity.

[32] The expected duration of each stage while the cycle runs its course, before it is interrupted by a new paradigm, is $T(n) = 1/[(N-n)b(n)m(n)]$.

[33] The measure of real output used here is the sum of production levels, $\Sigma_i x_i(t)$. Alternative measures such as consumption spending deflated by a price index, or the temporal utility of the representative household, give very similar results (but see note 6, above).

Figure 1. Output over the innovation cycle

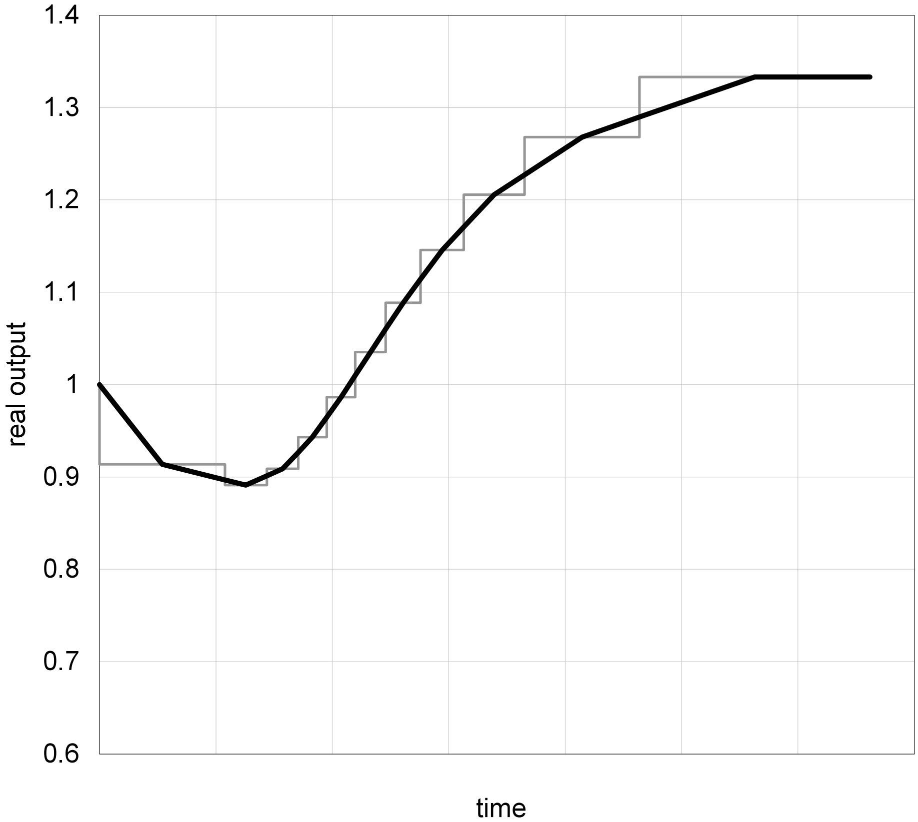

Figure 2. Wages and profits

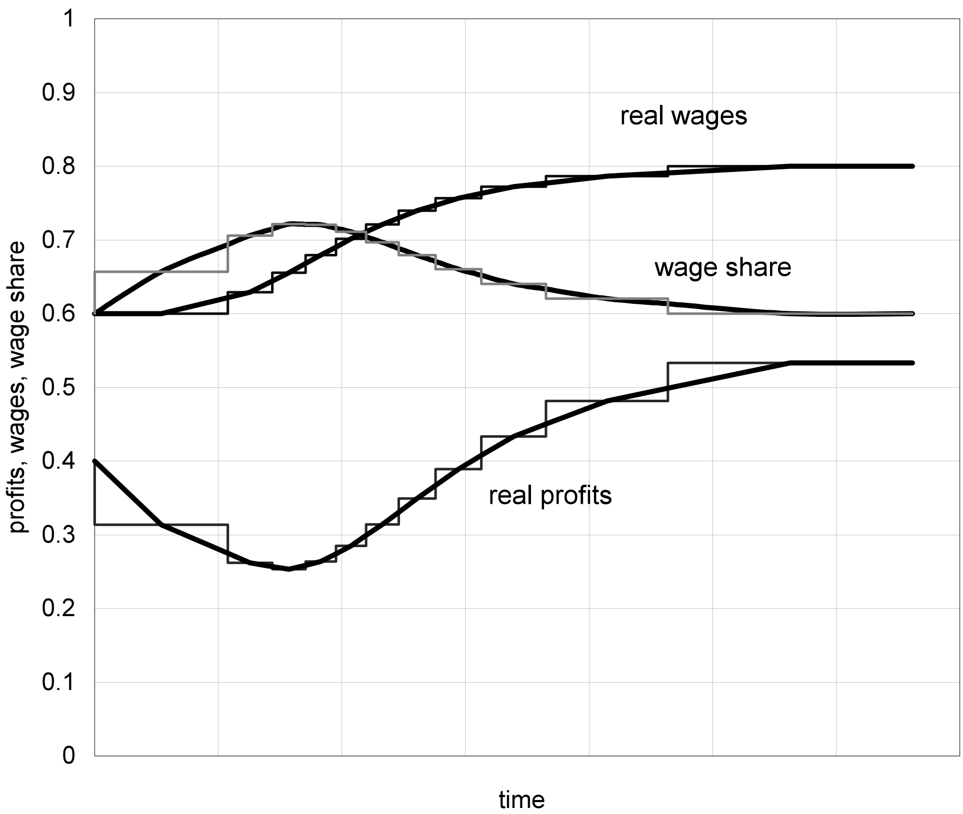

Figure 3. Diffusion of the new technology

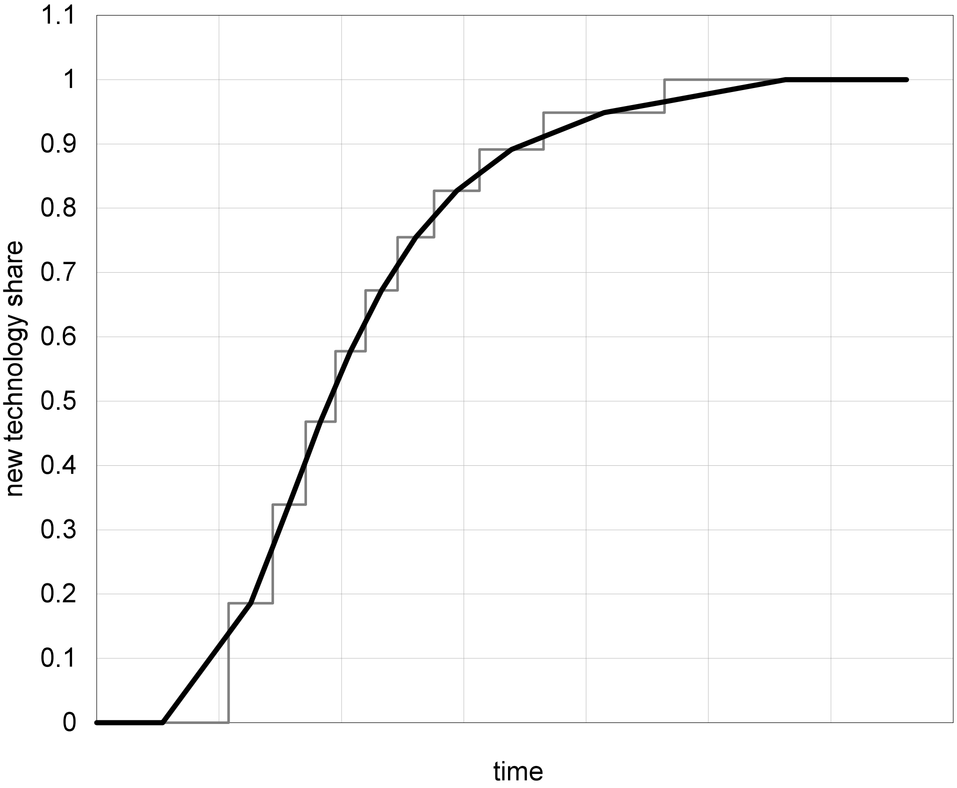

Figure 4. Market value of manufacturing and innovating firms