LERNER'S INDEX MEETS THE COASE CONJECTURE

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Abstract

Lerner's index, the markup of price over marginal cost, is the standard measure of monopoly power. Yet, as the Coase conjecture points out, its static context undermines its application to markets for durable goods, in which a monopolist's current sales "compete" with sales in later periods. The present paper integrates this dimension in a modified Lerner index, explicitly deriving the time consistent, profit maximizing, steady state monopoly markup under conditions of imperfect durability. This markup is decreasing in the elasticity of demand, the degree of durability and the monopoly's cost of credit, and increasing in consumers' credit costs.

JEL category: D42, C73

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1. Introduction

Lerner's index, the markup of price over marginal cost, is the standard measure of monopoly power used both in estimating the structure–conduct–performance paradigm of industrial economics (Weiss, 1974) and in gauging the impact of imperfect competition on cyclical fluctuations (Hall, 1986; Domowitz et al., 1988; Rotemberg and Woodford, 1999). Yet the static context in which Lerner's index is derived undermines its application to markets for durable goods, in which a monopolist's current sales "compete" with its sales in later periods. In the extreme case, as the Coase conjecture points out, perfect durability of a good can entirely countervail the market power of its monopoly seller—if trading is continuous and costless, and the monopoly is unable to commit to future prices or rent out the services of its product (Coase, 1972).[1] This essential difference between durables and non-durables may underlie significant differences in the cyclical behavior of their respective markups, pointed out by Domowitz et al. (1988).

This paper derives a time-consistent dynamic version of Lerner's index that integrates the intertemporal dimension of durable goods in the measurement of market power by modeling the implicit dynamic bargaining process between the monopoly and its market indicated by the Coase conjecture. However, while maintaining the basic assumptions of no precommitment to future prices and no rental market, it allows imperfect durability and discrete time intervals between price settings. Demand from the durable good derives explicitly from consumers' maximization of an intertemporal

[1] Formal analyses show that the Coase conjecture holds not only when its conditions are fully met but also in the limit, as the depreciation rate and the time between price changes become infinitesimally small (Stokey, 1981; Bond and Samuelson, 1984; Gul et

utility function defined as the discounted sum of temporal utility streams which stem from current holdings of the stock of a durable good and current consumption of a non-durable good.[2] Purchases of these goods are financed by an exogenous income stream, which consumers can shift from period to period through access to a capital market. Consumer demand is then determined in each period on the basis of current prices and holdings of the durable good, and expectations of future prices. The non-durable good is competitively supplied while the durable good is supplied by a single seller. As consumers are numerous and diffuse, the monopolist plays the role of Stackelberg leader in this dynamic pricing process: it sets its markup so as to maximize the net present value of its profits, anticipating consumers' reactions to its current and future prices while unable to commit to future prices.[3] In steady state equilibrium we require that all expectations are consistent—consumers' anticipations of monopoly prices, and the monopoly's anticipation of consumer behavior—and that prices and stocks are stable.

Specific functional forms are then used to derive an explicit expression for the steady state markup. It is equal to the inverse of the elasticity of demand, to which the markup is equal in static monopoly, multiplied by a factor that decreases in the durability of the good and in the monopoly's cost of credit, and increases in consumers' cost of credit. This accords with the analogy that Gul et al. (1986) draw between the Coase conjecture and Rubinstein's (1982) dynamic bargaining model. As Rubinstein's

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al., 1986).

[2] We use a stock adjustment model, of the type specified by Houthakker and Taylor (1970) in their comprehensive study of consumer demand in the United States.

[3] This is a "closed-loop" equilibrium; as the monopoly is the only strategic agent in the market it is subgame perfect.

model ties the division of payoffs between agents to their relative rates of time preference,[4] so do credit cost differentials between the monopoly seller of an imperfectly durable good and the buyers of the good affect their relative intertemporal bargaining power. Tighter credit conditions for seller or buyers restrict their respective flexibility to shift sales or purchases over time, lowering or raising the steady state markup.

Finally, numerical analysis indicates the magnitude of the different effects for typical parameter values. The degree of durability, measured as the inverse of the depreciation rate, has a large effect on the markup—under the terms of the Coase conjecture, which allows no precommitment to future prices, or rental of the services provided by the durable good. Variation in the markup that derives from the disparity between consumers' and producers' discount rates increases in relative terms as the degree of durability increases, although it is relatively small for typical fluctuations in the interest rate variables.

The structure of the paper is as follows. Section 2 describes the basic elements of the model and derives consumer demand as a function of current and anticipated prices. Section 3 then solves the monopoly's profit maximizing price schedule and characterizes the steady state price it sets in equilibrium. Section 4 provides numerical values of the steady-state markup for typical values of the depreciation rate consumers' and producers' discount rates. and Section 5 concludes.

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[4] Gul et al. (1986) assume symmetric credit conditions.

2. Demand

Consider an economy populated by a large number of identical, infinitely–lived households. Each household i derives utility in period t from consumption of a non–durable good, z_{it}, which is consumed in the same period it is purchased, and from its stock of a durable good, s_{it}. Denote household i's temporal utility by $u(s_{it}, z_{it})$. Assume that it is increasing and concave in each argument, its cross elasticity is positive (the durable and nondurable goods are complements), and its Hessian matrix is negative semi–definite; denoting partial derivatives by subscripts, $u_s, u_z > 0$, $u_{ss}, u_{zz} \leq 0$, $u_{sz} > 0$, and $u_{ss}u_{zz} - (u_{sz})^2 \geq 0$. Time is discrete, with the fixed length of each period reflecting exogenous transaction costs. Assume household utility is additively separable over time, discounted by a constant factor of γ per period. The discounted sum of the representative household's utility over its infinite horizon is then

$$U_i = \sum_{t=1}^{\infty} \gamma^t u(s_{it}, z_{it}) \tag{1}$$

We take the initial stock of the durable good s_{i0} as given, and let x_{it} denote the quantity of the durable good purchased by household i in period t. Denoting by δ the fraction of the stock that depreciates in each period, and setting $\alpha = 1 - \delta$, we have for $t = 1, 2, \ldots$

$$s_{it} = \alpha s_{it-1} + x_{it} \tag{2}$$

Each household earns income y_{it} in period t, and is assumed to have access to a credit market where it can borrow or lend at the interest rate r_c; denote the associated discount factor by $\beta = 1/(1+r_c)$. We assume the non–durable good is competitively supplied, and use its price as our numeraire, and denote the price of the durable good in period t by p_t. The large number of households implies that each individual household assumes it has no impact on producer behavior, and therefore acts without strategic considerations.

Consider the optimization problem of the representative household (dropping the household index for notational convenience). Denote the current and future price sequence it anticipates by $\{p_t\}$, and its income stream by $\{y_t\}$. The household's objective is then to find $\{x_t\}$ and $\{z_t\}$ that maximize (1) subject to:

$$\sum_{t=1}^{\infty} \beta^t [y_t - (p_t x_t + z_t)] = 0 \tag{3}$$

where s_0 is given, and s_t is defined by (2) for $t \geq 1$.

Forming the Lagrangian, L, and denoting the shadow price of the budget constraint by λ, first–order conditions are: [5]

$$\partial L / \partial z_t = \gamma^t u_z(s_t, z_t) - \lambda \beta^t = 0 \tag{4}$$

$$\partial L / \partial x_t = \sum_{k=1}^{\infty} \gamma^k u_s(s_k, z_k) \partial s_k / \partial x_t - \lambda \beta^t p_t = 0 \tag{5}$$

for $t =$ 1, 2, … From this it follows that (a detailed derivation is given in the Appendix):

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[5] Our assumptions on u ensure that second-order conditions also hold.

$$u_s(s_t, z_t) \ = \ \lambda\,(\,\beta\,/\,\gamma\,)^{\,t}\,(\,p_t - \alpha\,\beta\,p_{t+1}\,) \qquad (6)$$

Combining equations (4) and (6), we find that in each period the consumer equates the marginal rate of substitution between the durable and non–durable good to a function of current and next–period prices:

$$u_s(s_t, z_t)\,/\,u_z\,(s_t, z_t) \ = \ p_t - \alpha\,\beta\,p_{t+1} \qquad (7)$$

The comparative statics of demand are then readily derived by total differentiation of (7). As the left-hand side is declining in the stock of the durable good, current demand for the durable good is negatively related to its current price, p_t, and to the stock of the durable good held by the consumer at the beginning of the period, $s_{t\text{-}1}$. Its is positively related to its anticipated price in the next period, p_{t+1}, and to β = $1/(1+r_c)$, hence negatively related to the cost of consumer credit, r_c .

3. Monopoly pricing

We now derive the optimal pricing policy for a monopoly faced with such demand. To remove the income effect of changes in the interest rate on demand for the durable good and thus highlight its price effect we assume that utility is quasi-linear, and that the cost of consumer credit equals the representative household's rate of time preference, *i.e.*, $\gamma = \beta$. Specifically we assume that temporal utility has the form:

$$u(s\,,\,z) \ = \ s^{1-1/\sigma}/(1 - 1/\sigma) \ + \ z \qquad (8)$$

from which it follows that the desired stock of the durable good is

$$s_t^* = s_t(p_t, p_{t+1}) = (p_t - \alpha\beta p_{t+1})^{-\sigma} \tag{9}$$

for $t \geq 1$. Current demand is then:

$$x_t(p_t, p_{t+1}, s_{t-1}) = s_t^* - \alpha s_{t-1} = (p_t - \alpha\beta p_{t+1})^{-\sigma} - \alpha s_{t-1} \tag{10}$$

Now let r_m be the monopoly's cost of capital and let $\rho = 1/(1+r_m)$. Assume the variable unit cost of producing the durable good is constant, and denote it by c.[6] Then in each period t the monopoly sets its price p_t by maximizing its discounted profit stream from that point on, finding values $\{p_i\}_{i \geq t}$ that solve:

$$\max \pi_t = \sum_{i=t}^{\infty} \rho^{t-i}(p_i - c)x_i \tag{11}$$

where the $\{x_i\}_{i \geq t}$ are defined by (10), s_{t-1} is given, and s_i is defined by (1) for $i \geq t$.

Then, for a given initial stock level, s_0, and income stream $\{y_t\}_{t \geq 1}$, an equilibrium is a sequence of prices $\{p_t\}$, purchases of the durable good $\{x_t\}$, purchases of the non–durable good $\{z_t\}$, and stock levels of the durable good, $\{s_t\}$, such that at each time t:

- The sequence of future purchases $\{x_i\}_{i \geq t}$ and $\{z_i\}_{i \geq t}$ solves (3), maximizing household utility subject to the budget constraint implied
- stock levels and purchases satisfy for any t, $\{x_i\}_{i \geq t}$, maximizes U given $\{p_i\}_{i \geq t}$, π_t,

[6] A possible effect of interest rates on variable production costs is suppressed to highlight their effect on intertemporal market power.

and the purchases and stocks satisfy (1), (11), and (12).

After substitution we have:

$$\pi_t = (p_t - c)[(p_t - \alpha\beta p_{t+1})^{-\sigma} - \alpha s_{t-1}] + \rho(p_{t+1} - c)[(p_{t+1} - \alpha\beta p_{t+2})^{-\sigma} - \alpha(p_t - \alpha\beta p_{t+1})^{-\sigma}] + \ldots \qquad (12)$$

Differentiating this with respect to p_t gives the monopoly's first-order condition:[7]

$$\partial \pi_t / \partial p_t = (p_t - \alpha\beta p_{t+1})^{-\sigma} - \alpha s_{t-1} - (1/\sigma)(p_t - c)(p_t - \alpha\beta p_{t+1})^{-\sigma-1}$$
$$+ (\alpha\rho/\sigma)(p_{t+1} - c)(p_t - \alpha\beta p_{t+1})^{-\sigma-1} = 0 \qquad (13)$$

In a steady state in which prices and stocks are stable this implies that:

$$(p_s - c)/p_s = (1/\sigma)(1-\alpha)(1-\alpha\beta)/(1-\alpha\rho) \qquad (14)$$

where p_s denotes the steady state price of the durable good. Recalling that $\beta = 1/(1+r_c)$, $\rho = 1/(1+r_m)$ and $1-\alpha = \delta$, we can write:

$$(p_s - c)/p_s = \frac{1}{\sigma}\cdot\frac{\delta + r_c}{\delta + r_m}\cdot\frac{1+r_m}{1+r_c} \qquad (15)$$

Equation (15) highlights the different effects of the parameters of the model. As in Lerner's index, the markup varies inversely with the elasticity of demand, σ, which determines the seller's temporal monopoly power. But it also varies inversely with the

[7] Our assumptions ensure that the second-order condition also holds.

degree of durability (measured as the inverse of the rate of depreciation), as implied by the Coase conjecture. In addition, when consumers and seller face different credit conditions, the markup varies directly with consumer credit cost and inversely with producer credit cost, as implied by the analogy to Rubinstein's (1982) bargaining result.

4. Numerical Analysis

Numerical analysis of equation (15) indicates the magnitude of the different effects for typical parameter values. These are indicated in Table 1, in which we hold elasticity of demand equal to 1.5 (it enters multiplicatively in (15); table entries for other values would vary proportionately), and have the depreciation rate vary across its columns. The interest rates facing consumers and producers, which vary across table rows, are computed from actual interest rates faced by consumers and producers over the period 1972-97. The bank rate for 48-month automobile loans to consumers serves as a proxy for the nominal consumer interest rate, and Moody's Aaa corporate bond rate serves as our measure of the nominal cost of credit to producers. Both are converted to real rates by deducting the annualized current 3-month Treasury bill rate and adding the average real return on 3-month Treasury bills over the entire period (1.4%).

The degree of durability, measured as the inverse of the depreciation rate, has a large effect on the markup—under the terms of the Coase conjecture, which allows no precommitment to future prices, or rental of the services provided by the durable good. Variation in the markup that derives from the disparity between consumers' and producers' discount rates increases in relative terms as the degree of durability increases, although it is relatively small for typical fluctuations in the interest rate variables.

5. Concluding Remarks

This paper extends Lerner's index of monopoly power for durable goods, accounting for the effect highlighted by the Coase conjecture, that in such markets a monopolist's current sales "compete" with its sales in later periods—if it is unable to commit to future prices or rent out the services of its product. It derives a time-consistent dynamic version of Lerner's index that integrates the intertemporal dimension of durable goods in the measurement of market power by formulating the implicit dynamic bargaining process indicated by the Coase conjecture under "normal conditions" of imperfect durability and discrete time intervals between price settings. Specific functional forms are then used to derive an explicit expression for the steady state markup, in which the inverse of the elasticity of demand, to which the markup is equal in static monopoly, is multiplied by a factor that decreases in the durability of the good and in the monopoly's cost of credit, and increases in consumers' cost of credit. This highlights the analogy between the Coase conjecture and Rubinstein's (1982) dynamic bargaining model: credit cost differentials between the monopoly seller of an imperfectly durable good and the buyers of the good affect their relative intertemporal bargaining power. Tighter credit conditions for seller or buyers restrict their respective flexibility to shift sales or purchases over time, lowering or raising the steady state markup.

This modified version of the Lerner index provides a convenient basis for distinguishing between durables and non-durables both in regard to the links between market structure, conduct and performance, and in the cyclical behavior of markups.

Moreover, it suggests that credit cost differentials between consumers and producers may be significant in markets for durable goods, although our numerical results indicates that these effects are generally weak; identifying their role may require careful control of the many other factors that affect industry markups.

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Appendix: Derivation of equation (6).

From recursive application of equation (2) we have

$$s_t = \sum_{j=0}^{t} \alpha^{t-j} x_j + \alpha^t s_0$$

Hence we can write the first-order conditions thus:

$$\partial \mathcal{L} / \partial x_t = \sum_{i=t}^{\infty} \gamma_i \alpha^{i-t} u_s(s_i, z_i) - \lambda \beta^t p_t = 0 \qquad \text{(A1)}$$

and

$$\partial \mathcal{L} / \partial x_{t+1} = \sum_{i=t+1}^{\infty} \gamma^i \alpha^{i-(t+1)} u_s(s_i, z_i) - \lambda \beta^{t+1} p_{t+1} = 0. \qquad \text{(A2)}$$

Subtracting (A2) from (A1) and rearranging terms gives equation (6).

Table 1. Steady-state markups

Year	*consumers' interest rates* (r_c)	*producers' interest rates* (r_m)	Annual depreciation rate							
			100%	50%	20%	10%	5%	3%	2%	1%
1972	7.2%	4.5%	66.7%	34.1%	14.5%	7.7%	4.2%	2.7%	1.9%	1.0%
1973	4.4%	1.8%	66.7%	34.1%	14.6%	7.9%	4.5%	3.0%	2.2%	1.3%
1974	4.3%	2.0%	66.7%	34.0%	14.4%	7.7%	4.3%	2.8%	2.0%	1.1%
1975	6.7%	4.3%	66.7%	34.0%	14.3%	7.6%	4.1%	2.6%	1.8%	1.0%
1976	7.3%	4.7%	66.7%	34.1%	14.4%	7.6%	4.1%	2.6%	1.8%	0.9%
1977	6.8%	4.0%	66.7%	34.1%	14.5%	7.8%	4.2%	2.7%	1.9%	1.0%
1978	5.0%	2.8%	66.7%	34.0%	14.3%	7.6%	4.2%	2.7%	1.9%	1.0%
1979	3.2%	1.0%	66.7%	34.0%	14.4%	7.8%	4.4%	3.0%	2.3%	1.4%
1980	3.9%	1.8%	66.7%	34.0%	14.3%	7.7%	4.3%	2.8%	2.0%	1.2%
1981	3.6%	1.5%	66.7%	34.0%	14.3%	7.7%	4.3%	2.9%	2.1%	1.2%
1982	7.0%	4.2%	66.7%	34.1%	14.5%	7.8%	4.2%	2.7%	1.9%	1.0%
1983	6.3%	4.6%	66.7%	33.8%	14.1%	7.3%	3.9%	2.4%	1.7%	0.9%
1984	5.4%	4.5%	66.7%	33.6%	13.7%	7.0%	3.6%	2.2%	1.5%	0.8%
1985	6.5%	5.1%	66.7%	33.8%	13.9%	7.2%	3.8%	2.3%	1.6%	0.8%
1986	6.5%	4.3%	66.7%	34.0%	14.2%	7.5%	4.0%	2.5%	1.8%	0.9%
1987	5.8%	4.8%	66.7%	33.6%	13.7%	7.1%	3.6%	2.2%	1.5%	0.8%
1988	5.3%	4.3%	66.7%	33.6%	13.8%	7.1%	3.7%	2.3%	1.5%	0.8%
1989	5.1%	2.5%	66.7%	34.1%	14.5%	7.9%	4.4%	2.9%	2.1%	1.1%
1990	5.4%	3.1%	66.7%	34.0%	14.3%	7.7%	4.2%	2.7%	1.9%	1.0%
1991	6.9%	4.6%	66.7%	34.0%	14.3%	7.5%	4.0%	2.5%	1.8%	0.9%
1992	7.1%	6.0%	66.7%	33.6%	13.8%	7.1%	3.6%	2.2%	1.5%	0.8%
1993	6.4%	5.5%	66.7%	33.6%	13.7%	7.0%	3.6%	2.2%	1.5%	0.7%
1994	5.1%	5.0%	66.7%	33.4%	13.4%	6.7%	3.4%	2.0%	1.4%	0.7%
1995	5.3%	3.4%	66.7%	33.9%	14.2%	7.5%	4.0%	2.5%	1.8%	0.9%
1996	5.3%	3.7%	66.7%	33.8%	14.0%	7.3%	3.9%	2.4%	1.7%	0.9%
1997	5.2%	3.5%	66.7%	33.8%	14.1%	7.4%	3.9%	2.5%	1.7%	0.9%

Source: Interest rates, bond yields and CPI from Statistical Abstract of the United States, various years, and authors' calculations (see text for details)